UNIVERSITY OF CALIFORNIA, SAN DIEGO

Experiments on Viscous and Asymmetry-Induced Transport in Magnetized, Pure Electron Plasmas

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Physics

by

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1999
The dissertation of Jason Michael Kriesel is approved, and it is acceptable in quality and form for publication on microfilm:

Chairman

University of California, San Diego

1999
This thesis is dedicated to

MiChelle,

the best friend a man could hope for;

and also to

Bob,

the best friend an experimentalist could hope for.
Perhaps, if I am very lucky, the feeble efforts
of my lifetime will someday be noticed,
and maybe, in some small way,
they will be acknowledged as
the greatest works of genius ever created by Man.

– Jack Handey
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Vita, Publications and Fields of Study

Vita

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<td>26 October 1969</td>
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<td>1994</td>
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<td>1999</td>
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Publications


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Fields of Study

Major Field: Physics

Studies in Plasma Physics
Professors Daniel Dubin and Tom O’Neil

Studies in Mechanics
Professor Herbert Levine

Studies in Electromagnetism
Professor Tom O’Neil

Studies in Quantum Mechanics
Professor George Fuller and Marvin Goldberger

Studies in Statistical Mechanics
Professor Daniel Dubin

Studies in Galactic Dynamics
Professor Patrick Diamond

Studies in Mathematical Physics
Professor Roger Dashen

Studies in Space Shuttle Operations
Professor Sally Ride
Abstract of the Dissertation

Experiments on Viscous and Asymmetry-Induced Transport in Magnetized, Pure-Electron Plasmas

by

Jason Michael Kriesel
Doctor of Philosophy in Physics
University of California, San Diego, 1999
Professor C. Fred Driscoll, Chairman

Experiments are presented on two different types of cross-magnetic-field transport in pure-electron plasma columns: “asymmetry-induced transport” which is a radial expansion of the plasma column due to azimuthally asymmetric electric or magnetic fields, leading to bulk expansion and particle loss; and “viscous transport” which is a rearrangement of particles due to internal like-particle interactions, leading to the confined global thermal equilibrium state.

Experiments on asymmetry-induced transport identify two different transport regimes, “slightly-rigid” and “highly-rigid”. Here the plasma rigidity, \( R \equiv \frac{f_b}{f_E} \), is the ratio of the axial bounce frequency to the azimuthal \( E \times B \) rotation frequency. In the slightly-rigid regime (1 < \( R < 10 \)), the transport scales as \( V_a R^{-2} \), where \( V_a \) is the applied asymmetry strength. The \( V_a^{1/2} \) scaling is a direct contradiction to most current theories, which predict either a \( V_a^{1/2} \) or \( V_a^{2} \) scaling. The \( R^{-2} \) scaling has previously been observed for the plasma loss rate due to inherent trap asymmetries (also known as anomalous transport) on 5 different machines.
This “slightly-rigid” mechanism appears to “turn-off” as the rigidity is increased into the range $\mathcal{R} = 10$ to 20. In the highly-rigid regime ($\mathcal{R} > 20$), the transport scales as $V_a^2 \mathcal{R}^0$. This scaling is consistent with a previously studied transport mechanism known as “rotational-pumping”, where the axial energy is pumped due to asymmetries in the effective plasma length.

The measured viscous transport is observed to be proportional to the shear in the total ($\mathbf{E} \times \mathbf{B} +$ diamagnetic) fluid rotation of the plasma for both hollow and monotonic rotation profiles. I determine the local viscosity coefficient $\eta$ in the plasma from measurements of the local flux of electrons. The measured viscosity is $50 - 10^4$ times larger than expected from classical transport due to short-range velocity-scattering collisions, but is within a factor of 10 agreement with recent theories by O’Neil and Dubin of transport due to long-range drift collisions. The measured viscosity scales with magnetic field $B$ and length $L$ roughly as $\eta \propto B/L$. This scaling suggests a finite length enhancement of the viscosity, which occurs because particles interact many times as they bounce axially before they are sheared apart azimuthally.
Chapter 1

Introduction and Summary

1.1 Introduction

Quiescent plasmas of only one sign of charge (e.g. pure-electron plasmas) are readily confined in simple cylindrical traps. These (completely) non-neutral plasmas provide excellent laboratory systems for measuring the transport of particles [1], energy [39], and momentum [16] across a magnetic field. In this thesis I present detailed measurements on two different types of cross-field transport in pure-electron plasmas: “asymmetry-induced transport” which is a radial expansion of the plasma column due to asymmetric external electric or magnetic fields, leading to the eventual loss of particles from the trap; and “viscous transport” which is a rearrangement of the plasma due to internal shear-driven interactions, leading to the confined global thermal equilibrium state.

Asymmetry-induced and viscous transport studies are useful in understanding two important properties of single-signed plasmas: long-time confinement and a confined global thermal equilibrium state, both of which are attainable with static trapping fields (unlike neutral plasmas). These two unique properties make non-neutral plasmas attractive for a variety of experiments and applications. Studies of basic plasma phenomena are not limited to transport, but also include plasma
waves [11]; Debye shielding [36]; and one-component-plasmas, liquids, and crystals [44]. Applications which are being investigated include the trapping of anti-hydrogen [34], ion clocks [60], pressure standards [49], and ion-mass spectroscopy [58].

Asymmetry-induced transport leads to the degradation of the excellent confinement properties of non-neutral plasmas. In cylindrical traps, a magnetic field is applied along the axis of the trap. The plasma rotates about this axis, which leads to a confining Lorentz force directed radially inward. The confinement is limited by unintentional trap asymmetries which drag on the rotating plasma causing radial expansion and particle loss across the magnetic field. Recently a method know as the “rotating wall”[40] has been devised to counteract this expansion. This method works by applying an additional asymmetry to the trap walls that varies in time, and in effect spins faster than the plasma.

The mechanism by which asymmetric fields interact with a non-neutral plasma is currently not well understood. With the exception of so-called “rotational-pumping” [6, 8], theoretical treatments [45, 7] have had little success in describing the most basic measurements. Previous experiments have measured the radial transport due to inherent trap asymmetries on many different machines over a span of 20 years [15, 14, 5, 25], and some experiments have been done on applied asymmetries [28, 50, 40]. In all this experimental work, a coherent picture of asymmetry-induced transport has been lacking. In this thesis, I present simple experiments on the transport induced by both applied and inherent field asymmetries. The measurements exhibit clear scaling laws, which tie together much of the previous experimental observations, and lay a detailed empirical foundation for future theoretical work.

Viscous transport in a non-neutral plasma comes about due to shears in
the rotation. While asymmetry transport is due to an external field acting on the plasma, viscous transport is due to the plasma acting upon itself. In this case, internal like-particle interactions leads to the relaxation of the plasma towards a confined global thermal equilibrium state. In equilibrium, the plasma is quiescent and easily controlled, and it is even possible to apply the powerful tool of thermodynamics to some aspects of the plasma behavior [23]. Viscous transport is not only of interest as a basic transport process, but also knowledge of the coefficient of viscosity is useful for studies of two-dimensional fluid dynamics using pure-electron plasmas [13, 48, 41, 33].

Previous experiments measured global rates toward thermal equilibrium, and found the relaxation occurred at a surprisingly high rate [16]. However, these measurements were not of sufficient detail to determine the viscosity in the plasma, nor were they able to confirm whether viscous forces were responsible for the observed transport. In this thesis, I present measurements that unambiguously identify the relaxation of the plasma to be due to viscous effects. In addition, I experimentally measure the coefficient of viscosity and make accurate comparisons to theories of transport due to like-particle interactions.

A result that is common to both asymmetry and viscous transport is that finite length effects appear to alter the cross-field transport of the electrons. Interestingly, the effects are opposite in the two different types of transport. For asymmetry transport a decrease in the plasma length leads to weaker transport as the column becomes more rigid, whereas for viscous transport a shorter plasma leads to an increase in transport as interacting electrons effectively collide more often.
1.2 Summary

1.2.1 Background

The transport experiments described in this thesis were performed using two different Penning-Malmberg traps, known by the acronyms EV [42] and CamV [2]. These devices are described in Chapter 2. They are similar in design, consisting of a stack of conducting cylindrical electrodes immersed in an axial magnetic field. The axis of the cylinder (and thus the direction of the magnetic field) defines the z-axis of a cylindrical coordinate. Negative voltages on the end cylinders provide complete axial confinement of the electron plasma, and the magnetic field gives radial confinement. The main diagnostic is a destructive dump of the electron column onto a collection device, from which I obtain the radial density profile of the plasma. Since the initial conditions are highly reproducible, I can deduce the time evolution of the plasma by holding different (but nearly identical) plasmas for longer and longer periods of time.

The basic motion of electrons in a Penning-Malmberg trap is also described in Chapter 2. Due to their thermal energy, electrons bounce back and forth in the z direction between the confining end potentials at the approximate rate $f_b$. The electrons also $\mathbf{E} \times \mathbf{B}$ drift in the azimuthal direction at the rate $f_E$, due to the strong radial electric field of the (completely un-neutralized) electron column. The ratio of the bounce rate to the rotation rate is defined to be the "rigidity" of the plasma, $\mathcal{R} \equiv \frac{f_b}{f_E}$. This dimensionless parameter is shown to be important in characterizing the transport properties of a non-neutral plasma.

1.2.2 Asymmetry Transport

I study asymmetry-induced transport by applying static voltages to sections of the trap wall, thus breaking the cylindrical symmetry of the trap; the
resulting radial expansion of the plasma column is then measured quantitatively. This induced transport has been measured for a wide range of applied voltage \( V_a \) and confining magnetic field \( B \) and for a range of plasma density \( n_p \), length \( L_p \), and temperature \( T \). The parameter dependence of the measured expansion rate is described by simple empirical formulas which primarily depend upon the magnitude of the applied voltage and the rigidity of the plasma, where \( \mathcal{R} \propto B T^{1/2} / n_p L_p \). Two different transport processes are observed depending on whether the plasma is “slightly-rigid” \( (1 < \mathcal{R} < 10) \) or whether the plasma is “highly-rigid” \( (\mathcal{R} > 20) \).

The electron column responds to an applied asymmetry by moving and distorting towards negative voltages on the wall. In general, I apply the voltage only over a portion of the plasma length, yet I observe that the entire column moves rigidly in a bounce-averaged sense. A simple force-balance model describes this rigid distortion over the entire range of plasma parameters, including when the plasma is only slightly rigid.

Surprisingly, in the slightly-rigid regime the asymmetry-induced expansion rate \( \Delta \nu_{(\rho z)} \) is accurately described by a simple empirical formula in which \( \Delta \nu_{(\rho z)} \propto V_a \mathcal{R}^{-2} \). The most robust (and most surprising) result is that asymmetry-induced transport increases linearly with the applied voltage. This simple voltage scaling directly contradicts current theories.

This so-called \( V_a \mathcal{R}^{-2n} \) transport mechanism not only causes plasma expansion due to an applied electric asymmetry, but also appears to be the cause of transport due to inherent trap asymmetries and due to applied magnetic asymmetries. All processes are observed to decrease with rigidity, scaling approximately as \( \mathcal{R}^{-2} \) on at least 5 different containment devices. However, there is presently no theoretical explanation for this seemingly ubiquitous transport mechanism. The detailed measurements presented here may provide guidance for much-needed sim-
ulations and theory development.

In the highly-rigid regime, the observed transport scales differently with both applied voltage and rigidity; in this regime the asymmetry-induced expansion rate scales as \( \Delta \nu_{(r^2)} \propto V_a^2 \mathcal{R}^0 \). The “\( V_a \mathcal{R}^{-2} \)” mechanism, which is dominant in the slightly rigid regime, appears to “turn off”, allowing a different mechanism to dominate. Expansion rate measurements in the highly-rigid regime are generally consistent with transport due to the so-called “rotational pumping” mechanism, which has been studied previously [8, 6]. However, there is some disagreement between the parameter scaling of the measurements and predictions from simple rotational pumping theory. A more detailed comparison awaits theory calculations appropriate for the experiments.

1.2.3 Viscous Transport

For studies of viscous transport, I minimize the transport due to external asymmetries, and deduce the viscosity in the plasma from measurements of the local particle flux. A local model of viscous transport, where particles move radially to reduce shears in the plasma rotation, is found to provide an excellent qualitative description of the local measurements. The measurements presented in Chapter 4 are of sufficient precision that I determine the local coefficient of viscosity in the plasma, allowing for detailed comparisons to theories of transport due to like-particle interactions. I find that this transport is essentially the same for plasmas with hollow rotation profiles as for plasmas with monotonic rotation profiles, contrary to previous theoretical predictions [20].

The calculated coefficient of viscosity is observed to be up to 4 orders of magnitude greater than predicted by \textit{Classical} velocity-scattering theory [53]. This theory considers short-range collisions where electrons interact over a distance, \( \delta \),
that is less than or equal to the cyclotron radius, i.e. \( \delta \leq r_c \). A new class of Long-Range theories [52, 20, 22] by Dubin and O’Neil consider interaction distance as large as a Debye length \( \delta \leq \lambda_D \). These theories can give considerably larger transport since in general \( \lambda_D \gg r_c \) in a non-neutral plasma, and the viscosity coefficient scales as \( \delta^2 \).

Two theoretical versions of viscous transport due to long-range collisions are within a factor of 10 of the measurements. The three-dimensional (3D) version [52] agrees well with the measurements at low rigidity \( R \approx 1 \), but is about a factor of 10 smaller than the data at higher rigidity \( R \approx 10 \). On the other hand, a two-dimensional (2D) theoretical enhancement [22] predicts rates that are about a factor of 10 too large for the entire range of the data.

While no current theory satisfactorily agrees with my measurements, I present a simple empirical hybrid of the 3D and 2D Long-Range theories which accurately describe the measured viscosity coefficients. This empirical formula implies that electrons interact over a distance on the order of a Debye length, with an effective collision frequency proportional to the estimated average number of collisions for a pair of interacting electrons. The number of collisions roughly increases with rigidity, as electrons interact multiple times due to their bounce motion in a finite length plasma.
Chapter 2

Background

2.1 Overview

This chapter provides general background information on the two experimental devices EV and CamV. Both devices were used in studies of asymmetry-induced transport, but just EV was used for the viscous transport studies. In this chapter I also define measured and calculated quantities, and relate some basic properties of pure-electron plasmas to the experiments presented in this thesis.

Both EV and CamV belong to a class of cylindrical devices called Penning-Malmberg traps, which originated with the V machine [10] constructed at the University of California at San Diego (UCSD) in the early 1970’s. The “V” stands for voltage containment, and the traps at UCSD are known by an acronym ending with the letter “V”. The “E” in EV stands for “Equilibrium”, referring to the confined global thermal equilibrium state it was designed to observe; and the “Cam” in CamV stands for “Camera”, referring to its CCD camera diagnostic.

In Section 2.2 I give a general description of the physical characteristics of the two devices. They consist of a stack of hollow conducting cylinders enclosed within a cylindrical vacuum chamber. The schematics of the electrode stacks are shown in Figure 2.1. The vacuum chamber resides within the bore of a solenoid,
which produces a uniform magnetic field coaxial with the conducting cylinders. The magnetic field provides radial confinement, and negative potentials applied to end cylinders provide axial confinement. The basic designs of the machines are quite similar; however, one difference between the two machines is the magnetic field source. CamV uses a super-conducting solenoid used to attain fields as high as 10 kGauss, whereas EV uses a copper-wire solenoid limited to 500 Gauss.

The diagnostics and operation of the machines is described in Section 2.3. Data is taken in an inject/hold/dump cycle, and the main diagnostic involves a destructive axial dump of the electron column onto a collection device. The primary measurement is the density profile of the electrons. In EV, a one-dimensional radial (r) density profile is constructed over many data cycles, which relies on the high degree of azimuthal symmetry and shot-to-shot reproducibility in the plasma. In contrast, the camera diagnostic of CamV gives a complete two-dimensional (r, θ) density map with one dump of the plasma. In addition to measuring the density, I also measure the temperature of the plasma. EV has the advantage of being able to measure the temperature across the entire diameter of the plasma, whereas in CamV the temperature is only measured at the center (r = 0) of the plasma column.

Since the initial conditions are highly reproducible, I can deduce the time evolution of the plasma by holding different (but nearly identical) plasmas for longer and longer periods of time. Using the measured radial density profiles taken at two different times, I determine the local radial flux of electrons. Also, from the basic measurements of density and temperature, I calculate other plasma quantities (e.g. plasma length, Debye length). Some of these calculations are defined in Section 2.4 along with the notation used throughout the thesis.

In Section 2.5, I define and describe relevant rates of motion and equilibra-
tion rates in a pure-electron plasma. The important concept of plasma rigidity, $\mathcal{R}$, is introduced in this section defined as the ratio of the bounce frequency to the $E \times B$ drift frequency.

In the last two sections of this chapter, I provide background information on two important properties of a pure-electron plasma. In Section 2.6 I discuss the exceptional confinement properties of non-neutral plasma in relation to the canonical angular momentum, which depends upon the mean-square-radius of the plasma. Asymmetries cause expansion and eventual loss of the plasma by breaking the azimuthal symmetry of the trap. Assuming adequate confinement the plasma will relax to the global thermal equilibrium state of rigid rotation, which is described in Section 2.7.

At the end of this chapter, I list the range of plasma parameters for the experiments presented in this thesis.

## 2.2 Physical Characteristics

The two experimental devices, EV and CamV, are quite similar in their physical characteristics. In this section, I give a general description of the two devices, while pointing out their subtle differences. A list of the most important differences is given in Table 2.1. A more detailed description of the EV apparatus can be found in previous UCSD theses by Alan Hyatt [42], Kevin Fine [31], and Travis Mitchell [47]; and a more detailed description of the CamV apparatus can be found in the previous UCSD thesis by Ann Cass [2].

EV was constructed by Driscoll and Fine for the purpose of observing the confined thermal equilibrium state of a pure-electron plasma. For this reason, the design, fabrication, and assembly focused on reducing inherent magnetic and electric field asymmetries in order to increase the plasma confinement time. EV
has the smallest inherent trap errors compared to all 6 of the UCSD V machines, past and present. The diagnostics on EV are well-suited for measurements of radial particle transport, which is the subject of this thesis. The primary drawback with using EV is the limited range in magnetic field.

CamV was constructed by Fine, Cass, and Driscoll for the purpose of studying 2D fluid dynamics. Basically, the design of CamV was patterned after EV with two major differences: (1) A CCD camera diagnostic (instead of a collimator and collector) to obtain detailed pictures of the plasma evolution; (2) a superconducting solenoid (instead of a copper-wire solenoid) to obtain relatively high magnetic fields.

Both devices are maintained at ultra-high vacuum (UHV) with a neutral background gas pressure below $10^{-9}$ Torr. The EV apparatus has the extra features of water-cooled vacuum walls and a titanium sublimation pump. These features allow EV to be operated continually at pressures near $5 \times 10^{-11}$ Torr.

Each vacuum vessel resides in the bore of a solenoid, which provides a uniform magnetic field, $B_z$, along the axis of the trap. In both machines, additional $B_x$ and $B_y$ saddle coils are used to make small corrections, which align the magnetic field axis with the axis of the cylinders. Alignment of the EV magnetic field is achieved by maximizing the central density as described in Reference [31].

<table>
<thead>
<tr>
<th></th>
<th>EV</th>
<th>CamV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic Field</td>
<td>Copper Wire ($\leq 500$ G)</td>
<td>Superconducting ($\leq 10$ kG)</td>
</tr>
<tr>
<td>Sectored Cylinders</td>
<td>4-Sector</td>
<td>4-Sector &amp; 8-Sector</td>
</tr>
<tr>
<td>Density Diagnostic</td>
<td>Collimator &amp; Collector $Q(r)$</td>
<td>Phosphor &amp; CCD $Q(r, \theta)$</td>
</tr>
<tr>
<td>Temp Diagnostic</td>
<td>Beach Analyzer $T_{\perp}(r)$</td>
<td>Evaporation Analyzer $T_i(0)$</td>
</tr>
</tbody>
</table>

Table 2.1: Differences in the physical features and diagnostics of the two machines.
The CamV magnetic field is aligned with a similar technique, but instead of just maximizing the central density, the mean-square-radius is also minimized.

The electrode stacks for both devices are shown in Figure 2.1. The two stacks consist of a series of electrically isolated cylinders, some of which are divided into azimuthal sections. Each cylinder has an alpha-numeric code with the letter identifying the cylinder type, and the number identifying its position in the stack. The letter “G” stands for gate, “L” for long, “H” for half, and “S” for sectored.

The physical dimensions of the cylinders are quite similar for the two machines. The EV cylinders have an inner radius of 3.81 cm with lengths of 7.89 cm for the G and S cylinders and 4.09 cm for the H3 cylinder. The CamV cylinders have an inner radius of 3.50 cm with the following lengths: the G and S cylinders are 7.0 cm long, the H cylinders are 3.5 cm long, and the L2 cylinder is 14 cm long.

I use the azimuthal sectors of the S cylinders to apply asymmetries for the studies of asymmetry-induced transport in Chapter 3. Alternatively, they can be used to send and receive wave signals. Each sector is electrically isolated from the other sectors as well as from the frame in which they reside. EV has one 4-sectored cylinder (S6), while CamV has one 4-sectored (S4), and one 8-sectored cylinder (S7). The 4-sectored cylinders have 60° azimuthal sections and the 8-sectored cylinder has 25° azimuthal sections.

Axial confinement is provided by applying negative potentials, $V_c = -100$ Volts, to two cylinders at opposite ends of the trap, while grounding the remaining cylinders. The distance between the two confinement cylinders defines the confinement length, $L_c$. This length can be varied in discrete steps by applying the confinement voltage to different cylinders. The confinement length on EV is varied over the range $L_c = 7.89$ to 35.6 cm, and on CamV it is varied from $L_c = 7.0$ to 42.0 cm.
Figure 2.1: Electrode stacks for EV and CamV.
I obtain average plasma densities which range from about $10^6 - 10^7$ cm$^{-3}$. In both machines, electrons are produced from a directly heated spiral of tungsten wire. The center of the spiral is biased negatively with respect to ground to a potential $V_b = -5$ to $-40$ Volts. The EV source is heated with a direct current (DC) of about 10 Amps, which provides an $r^2$ potential variation on the cathode. The intent is for this potential to match the $r^2$ variation in the space-charge potential of a uniform density column of electrons. On CamV, the source is heated with an alternating current (AC) to prevent distortion from the continual $\mathbf{j} \times \mathbf{B}$ force. The heating current is about 11 Amps (rms) at 16 kHz. Extra electronic circuits clamp the current at an adjustable DC level during injection, so that the $r^2$ matching potential on the cathode will be the desired sign and level. In Reference [46] a simple model is shown to accurately predict both the magnitude and shape of the initial density profile from a cathode with an $r^2$ potential variation.

### 2.3 Data Taking Cycle and Diagnostics

Both EV and CamV are operated in an inject/hold/dump cycle, as shown schematically in Figure 2.2 for the EV device. The confining potentials, $V_c = -100$ Volts, are applied in a specific repeatable sequence to cylinders at opposite ends of the plasma. The two cylinders used for confinement are labelled by the names “inject gate”, corresponding to the cylinder at the source end of the machine, and the “dump gate”, corresponding to the one at the diagnostic end of the machine.

A data taking cycle proceeds as follows: (a)During injection, the inject gate is grounded for at least 500 $\mu$sec, while the dump gate is held at $V_c$. A steady-state electron column forms between the source and the dump gate [46]. (b) The plasma is trapped by ramping the inject gate to $V_c$ in about 200 $\mu$sec, and is held by this configuration of potentials for the duration of the experiment, typically
100 \mu\text{sec} \leq t_{\text{hold}} \leq 10 \text{sec}. (c) At the end of the cycle the dump gate is switched to ground in about 2 \mu\text{sec}, and the electrons stream along the magnetic field lines to a charge collection device.

### 2.3.1 Density Diagnostic

The main diagnostic for both machines is the $z$-integrated density profile of the plasma. In CamV a full two-dimensional $(r, \theta)$ image of the plasma density
is obtained in just one dump cycle, while in EV a one-dimensional radial density
profile is built up over many cycles.

The density of electrons in a CamV plasma is measured using a phosphor
screen and CCD camera. The dumped electrons are accelerated onto the phosphor
by a potential of $V_a = +15$ kV. Light generated by the luminescence of the phosphor
passes through a vacuum window and is focused onto a $512 \times 512$ CCD chip. The
intensity of the light is proportional to the number of collected electrons.

The basic measurement is thus the number of electrons in the plasma, along
a field line at a particular radius $r$ and angle $\theta$. Historically, this value is written as
$Q(r, \theta)$, which represents the charge within a small tube of the plasma, as shown
in Figure 2.2(c) in relation to the EV density diagnostic. The measured value of
$Q(r, \theta)$ can be related to the $z$-integral of the three dimensional plasma density,
$n_{p}^{3D}(r, \theta, z)$, as

$$Q(r, \theta) = -eA \int dz \ n_{p}^{3D}(r, \theta, z). \quad (2.1)$$

Here $-e$ is the charge of an electron and $A$ is the area of the collection device,
which in this case is the area of the plasma as “seen” by one pixel, $A = 7.1 \times 10^{-4}$
cm$^2$.

I define a confinement density, $n_c(r)$, from the theta-average of $Q(r, \theta)$ by
dividing by the length, $L_c$, of the confinement cylinders, as

$$n_c(r) \ = \ \frac{1}{eAL_c} \int \frac{d\theta}{2\pi} \ Q(r, \theta) \quad (2.2)$$

$$\quad = \ \int \frac{d\theta}{2\pi} \ \int \frac{dz}{L_c} \ n_{p}^{3D}(r, \theta, z).$$

An example of a CamV dump image is shown in Figure 2.3 along with the calcu-
lated confinement density.

The actual density, $n_{p}^{3D}(r, \theta, z)$, within the plasma is generally slightly
larger than the confinement density, $n_c(r)$, due to electrons being reflected be-
Figure 2.3: CamV dump image and the calculated $\theta$-averaged density profile.
fore reaching the confinement gate. In other words, the effective length of the plasma, $L_p(r)$, is somewhat less than the confinement length, $L_c$ (see Figure 2.6).

In EV, the radial density profile is measured over many shots, using a moveable collimator plate and collector as shown in Figure 2.1. Electrons passing through the hole in the collimator plate are collected on a Faraday cup with known capacitance to ground, and the induced voltage is proportional to $Q(r, \theta)$. By stepping the collimator hole across the plasma diameter and repeating the same inject/hold/dump cycle at each position, the radial profile of the confinement density, $n_c(r)$, is constructed as defined by Equation 2.2. In the case of the EV measurement, the effective pixel area is the area of the collimator hole $A_h = 7.9 \times 10^{-2} \text{ cm}^2$ (radius $R_h = 0.159 \text{ cm}$), and only two values of $\theta$ are scanned for each radial position, as the hole is rotated through an arc which transects the trap axis.

The accuracy of the EV density profile relies on the two following assumptions: the column is azimuthally symmetric, and highly reproducible from shot-to-shot. An example of an EV density profile is shown in Figure 2.4. $Q(r, \theta)$ is measured at discrete radial positions separated by $\Delta r \approx 0.075 \text{ cm}$. At each position, 6 separate inject/hold/dump cycles are repeated. The vertical separation of the bars in Figure 2.4 is the standard deviation for the average density of 6 different (but nearly identical) plasmas. The measured density profile has a high degree of side-to-side symmetry and a small amount of shot noise (typically $\delta n / n < 1\%$).

The total number of electrons in the plasma, $N_{tot}$, is measured separately from the $n_c(r)$ density measurement. CamV is equipped with a collector plate which can be rotated in front of the phosphor screen. By dumping the plasma onto the plate, $N_{tot}$ is obtained from the total collected charge. This signal is used to
Figure 2.4: Example of EV density and temperature data. The radial confinement density, \( n_c(r) = \sum_{i=1}^{2} -Q(r, \theta_i)/eAL_c \), and perpendicular temperature, \( T_\perp(r) \), profiles are built up over many repetitions of the same inject/hold/dump cycle as the collimator hole is stepped across the diameter of the plasma column. The horizontal bar labelled \( 2R_h \) represents the diameter of the collimator hole.

calibrate the \( n_c(r) \) measurement by the phosphor and CCD using the relationship,

\[
N_{tot} = L_c \int_0^{R_w} 2\pi r \, dr \, n_c(r). \tag{2.3}
\]

In EV, \( N_{tot} \) is measured at each dump of the plasma by measuring the charge collected by the collimator plate (in addition to the small amount which is collected by the faraday cup). Fine showed that a small fraction (\( \sim 3\% \)) of the electrons, which are nominally located on field lines passing through the collimator hole, do not actually pass through the collimator hole when dumped [31]. This is attributed to the finite size of the electron cyclotron orbit, so the effect is larger at
larger temperature. The total charge measured from the constructed radial profile is typically a few percent less than the total charge measured by the collimator plate. For the EV data, I use Equation 2.3 to define an overall calibration of $n_c(r)$ for each profile. In practice, this amounts to multiplying $n_c(r)$ by a factor equal to about 1.03.

### 2.3.2 Temperature Diagnostics

EV is equipped with a perpendicular velocity analyzer (commonly called a "magnetic beach" analyzer), which is used to determine the radial profile of the average thermal energy perpendicular to the magnetic field, $T_\perp(r)$ [42]. An example of EV temperature data is shown as the solid circular points in Figure 2.4. Temperatures are calculated by measuring $Q(r, \theta)$ as a function of a secondary magnetic field and a reflecting potential. About 80 separate machine cycles are needed to calculate the temperature at each radial position, so a high degree of shot-to-shot reproducibility is crucial. The error bars in Figure 2.4 show the estimated error in the calculation. Temperature measurements are relatively time-consuming, and therefore, are not taken at every radial position. In addition, the measurement are not performed at the extreme radial edge of the plasma, because the small electron density there makes the measurement unreliable.

In CamV, the parallel temperature, $T_\parallel(0)$ is measured only at the radial center using an "evaporative" technique [27]. For this measurement, the collector plate is rotated in front of the phosphor screen, and the dump voltage is slowly ramped to ground over a time ($\sim 10$ ms) which is long compared to the dump time for a density measurement ($\sim 1 \mu s$). During the slow ramp of the confinement potential, electrons with sufficient parallel energy escape the confinement region, and are collected by the end plate. The number of collected electrons vs ramped
voltage is digitized and fit to the tail of a Maxwellian distribution, which defines $T_\parallel$. Since the space charge potential is most negative at the center of the plasma, the electrons escape from there first. The diagnostic thus measures the temperature parallel to the magnetic field at the axial center of the plasma $T_\parallel(0)$. In practice, I take the average value of the temperature for 5 separated machine cycles, with a shot-to-shot deviation in the calculation that is typically $\delta T/T \approx 0.1$.

For both EV and CamV data I use the respective temperature measurements to obtain a single global value $T$, which describes the thermal energy of the entire plasma. This approximation is discussed in Section 2.5.2.

2.4 Calculated Quantities

All experimental measurements of transport and plasma parameters are calculated from the the basic data of $n_c(r)$ and $T$. Some derived quantities are global, like the expansion rate for asymmetry-induced transport, $\nu_{(x)}$; and some are local, like the kinematic viscosity, $\kappa_x(r)$. In this section, I describe some of the calculated quantities and define notation which is used throughout the thesis.

2.4.1 Particle Fluxes

In measurements of both asymmetry-induced and viscous transport, I consider the flux, $\Gamma$, of particles moving radially across the magnetic field lines. The definition for the particle flux is derived from the continuity equation,

$$\frac{\partial n(r)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r \Gamma(r) = 0. \quad (2.4)$$

The experimental flux of particles, $\Gamma_x(r)$, is calculated from the change in the confinement density, $\Delta n_c(r)$, between two plasmas. The two plasmas are created with the same initial conditions, but held for different times, $t_1$ and $t_2 =$
\[ t_1 + \Delta t. \] The experimental particle flux describes the radial flow of particles, and is defined as

\[ \Gamma_x(r) \equiv -\frac{1}{r} \int_0^r \frac{dr'}{r'} \frac{\Delta n_e(r')}{\Delta t}. \]  

(2.5)

The flux at the radial position of the wall measures the flow of particles out of the trap, and is proportional to the loss of the total number of electrons as

\[ \frac{\Delta N_{tot}}{\Delta t} = -2\pi R_w L_c \Gamma(R_w). \]  

(2.6)

For all the experiments presented in this thesis, \( \Delta N_{tot}/\Delta t = 0 \) to within the experimental accuracy of the measurements.

### 2.4.2 Plasma Parameters

#### Axial Dependence

I do not obtain direct experimental information about the \( z \)-dependence of the plasma from the axial dump. However, the \( z \)-dependence of the plasma density, \( \tilde{n}(r,z) \), and self-consistent space-charge potential \( \tilde{\phi}(r,z) \), are routinely reconstructed using a widely utilized Poisson code [3]. (In this thesis I use the symbol “\~” to denote a quantity which depends upon \( r \) and \( z \).)

To find \( \tilde{n}(r,z) \) and \( \tilde{\phi}(r,z) \), I iteratively solve Poisson’s equation by assuming that the electrons are well-described by a (local) Boltzmann distribution along each field line

\[ \nabla^2 \tilde{\phi}(r,z) = 4\pi e \, \tilde{n}(r,z) = 4\pi e \, \tilde{n}(r,0) \exp \left\{ \frac{e\tilde{\phi}(r,z) - e\tilde{\phi}(0,z)}{T} \right\}. \]  

(2.7)

A smoothed function is fit to the confinement density data \( n_c(r) \), and the calculation is done on a \( 64 \times 256 \) \((r,z)\) grid. The boundary conditions are determined by \( V_c \) and \( L_c \), and the measured \( n_c(r) \) gives the constraint

\[ n_c(r) = \int \frac{dz}{L_c} \, \tilde{n}(r,z). \]  

(2.8)
Radial Dependence

In this thesis, I am only concerned with radial transport and the radial variations in plasma parameters. I only use Poisson’s equation to determine the plasma length and to get a more accurate form for the density and rotation frequencies in the plasma. I remove the $z$-dependence of various quantities by performing a density-weighted $z$-average. For a generic quantity $\tilde{y}(r, z)$ this average is defined by the equation,

$$y(r) \equiv \frac{1}{n_c(r)} \int \frac{dz}{L_c} \hat{n}(r, z) \tilde{y}(r, z).$$  \hspace{1cm} (2.9)

As an example, consider the density of electrons in the plasma. As mentioned in the previous section, the confinement density $n_c(r)$ is most likely an underestimate of the actual density, since it is calculated from the confinement length, $L_c$. A more accurate, yet still one-dimensional, estimate of the density is given by

$$n_p(r) \equiv \frac{1}{n_c(r)} \int \frac{dz}{L_c} \hat{n}(r, z) \hat{n}(r, z).$$ \hspace{1cm} (2.10)

I refer to $n_p(r)$ as the plasma density, since it better represents the actual density in the plasma than does $n_c(r)$.

I then use $n_p(r)$ to define the radially dependent plasma length, as

$$L_p(r) \equiv L_c \frac{n_c(r)}{n_p(r)}.$$  \hspace{1cm} (2.11)

For a confining potential of $V_c = -100$ V and a plasma density of $n_p \approx 10^7 \text{ cm}^{-3}$, the calculated plasma length is typically about 3 to 5 cm less than the confinement length, i.e. $L_p \approx L_c - 4$ cm.

An example of the difference between the calculated plasma density, $n_p(r)$, and the confinement density, $n_c(r)$ is shown in Figure 2.5. The data is for a short EV plasma confined within a single ring of length $L_c = 7.89$ cm, with confining
Figure 2.5: The calculated plasma density $n_p(r)$ is larger than the simple confinement density, $n_c(r)$. In conjunction, the calculated plasma length, $L_p(r)$ is less than the confinement length $L_c$. The difference in magnitude is particularly large in this case, because the plasma is relatively short.

Voltages of $V_c = -100$ Volts. The confinement density $n_c(r)$ is the average of the two sides of the radial scan shown in Figure 2.4. The magnitude of the plasma density $n_p(r)$ is as much as twice $n_c(r)$.

Also in Figure 2.4 the plasma length, $L_p(r)$, is shown to be less than the confinement length, $L_c$. The plasma length also decreases with radius, because the magnitude of the space-charge potential is less at the edge of the plasma. The radial variation in length implies that the plasma is somewhat spheroidal in shape, as shown schematically in Figure 2.2(b). For longer plasmas, the change in magnitude is not as dramatic, but the gradient of the plasma length, $\partial L_p(r)/\partial r$, is approximately the same.
Global

To characterize a process averaged over the entire plasma, I perform a radial integral weighted by the number of electrons at each radial position. The radial average \( \langle y \rangle \) of a generic quantity \( y(r) \) is defined as

\[
\langle y \rangle = \frac{L_c}{N_{\text{tot}}} \int_0^{R_w} 2\pi r \, dr \, n_c(r) \, y(r).
\]  

(2.12)

Global parameters such as the average plasma density, \( \langle n_p \rangle \), and the average plasma length, \( \langle L_p \rangle \), are used throughout Chapter 3 to describe the expansion rate of the plasma.

From the radial average of the plasma length, \( \langle L_p \rangle \), I calculate the line density (number per unit length), \( N_L \), defined by the equation

\[
N_L = \frac{N_{\text{tot}}}{\langle L_p \rangle}.
\]  

(2.13)

And from the mean-square-radius \( \langle r^2 \rangle \) I calculate the effective plasma radius,

\[
R_p \equiv (2 \langle r^2 \rangle)^{1/2}.
\]  

(2.14)

2.5 Rates

In this section, I define many of the rates used throughout the thesis, and provide numerical estimates for a typical EV plasma with \( n_p = 10^7 \text{ cm}^{-3} \), \( B = 100 \text{ G} \), \( L_p = 10 \text{ cm} \), and \( T = 1 \text{ eV} \).

2.5.1 Rates of Motion

The fastest rate of importance is the cyclotron frequency of the electron motion around a magnetic field line,

\[
\Omega_c \equiv \frac{eB}{m_e c}
\]  

(2.15)

\[ \approx 2.8 \times 10^8 \text{sec}^{-1} \ (2\pi)[B/100 \text{ G}], \]
Figure 2.6: Top: A side view of the plasma showing the axial bounce motion of an electron. Bottom: An expanded end view showing the $\mathbf{E} \times \mathbf{B}$ drift of the same electron along with the cyclotron motion. (Note: figure is not to scale.)
where \( m_e \) is the mass of an electron. All other particle motions are slower than the cyclotron motion, making guiding center theories applicable.

Typically, the next fastest rate is the axial bounce motion of an electron between the confining potentials, as shown in Figure 2.6 and given by

\[
\begin{align*}
    f_b(r) &\equiv \frac{v_\parallel}{2 L_p(r)}, \\
    \bar{f}_b(r) &\equiv \frac{\bar{v}}{2 L_p(r)}.
\end{align*}
\]  

(2.16)  

(2.17)

The bounce rate for an individual electron depends upon its individual axial velocity \( v_\parallel \). Therefore, along any given field line the plasma has a distribution of bounce frequencies. The quantity \( \bar{f}_b \) is often used in calculations, and it represents the bounce frequency of an electron moving at the thermal velocity \( \bar{v} \equiv \sqrt{T/m_e} \).

Since the plasma is completely un-neutralized, there is a net radial electric field \( E_r(r) = -\partial \phi(r)/\partial r \neq 0 \) due to the space charge of the plasma. This leads to an \( \mathbf{E} \times \mathbf{B} \) drift in the azimuthal direction

\[
\omega_E(r) \equiv f_E(r)(2\pi) \equiv c \frac{\partial \phi(r)/\partial r}{r B} \quad (2.18)
\]

\[
\approx 1.4 \times 10^6 \text{sec}^{-1} (2\pi) \left[ \frac{n_p(r)}{10^7 \text{cm}^{-3}} \right] \left[ \frac{100 \text{G}}{B} \right],
\]

where the numerical approximation is written by assuming a constant density. For viscosity measurements (Chapter 4) the last line is not sufficiently accurate. In this case, I compute \( \tilde{\omega}_E(r, z) \) from \( \tilde{\phi}(r, z) \) and then calculate \( \omega_E(r) \) with a density-weighted \( z \)-average as defined in Equation 2.9.

I find that the ratio of the bounce frequency to the \( \mathbf{E} \times \mathbf{B} \) drift frequency is an extremely useful way to characterize a plasma. This ratio is known as the
plasma rigidity, $\mathcal{R}(r)$, defined by the equation

$$
\mathcal{R}(r) = \frac{f_b(r)}{f_E(r)} 
$$

\approx 1.46 \left[ \frac{B}{100 \text{ G}} \right] \left[ \frac{T}{1 \text{ eV}} \right]^{1/2} \left[ \frac{10^7 \text{ cm}^{-3}}{n_p(r)} \right] \left[ \frac{10 \text{ cm}}{L_p(r)} \right].

(2.19)

(Note: that according to the definition of the plasma length $L_p$ (Equation 2.11) the combination of $L_c$ and $n_c$ can be used in Equation 2.19 in place of the combination $L_p$ and $n_p$.) In Chapter 3, I use the average rigidity, $\langle \mathcal{R} \rangle$, to describe the plasma, which is calculated from Equation 2.19 using the average density $\langle n_p \rangle$ and length $\langle L_p \rangle$.

For large values of the rigidity, a thermal electron bounces between the ends many times before rotating about the axis once. In this case, theories often consider the electron to be a “rigid” bounced-average rod of charge. In the opposite limit (low values of $\mathcal{R}$), the electron is “floppy” and rotates many times before bouncing.

The cross-field transport rates are shown in the following chapters to depend upon the rigidity. For reference, I divide the range of the rigidity parameter into three categories:

- Floppy: $\mathcal{R} < 1$
- Slightly Rigid: $1 \leq \mathcal{R} \leq 10$
- Highly Rigid: $\mathcal{R} > 20$.

The range in rigidity $10 < \mathcal{R} < 20$ is found to mark a transition in the asymmetry-induced transport properties of the plasma, but is sadly not given a name.

### 2.5.2 Equilibration Rates

The rate at which electrons thermalize along any single field line is much larger than the cross-field transport rates presented in this thesis. Therefore, I
consider the electrons to be in thermal equilibrium along each field line in the remainder of this thesis, and make no distinction between $T_{||}$ and $T_{\perp}$.

Electrons quickly thermalize along the magnetic field lines due to collisions. The electron-electron collision frequency for 90° scattering collisions is given by

$$
\nu_{ee} \equiv \frac{16\sqrt{\pi}}{15} n \bar{v} b^2 \ln(r_c/b)
$$

$$
\approx 180 \text{ sec}^{-1} \left[ \frac{n_p}{10^7 \text{ cm}^{-3}} \right] \left[ \frac{1 \text{ eV}}{T} \right]^{3/2}.
$$

Here, $r_c \equiv \bar{v}/\Omega_c$ is the cyclotron radius and $b \equiv e^2/T$ is the distance of closest approach for thermal electrons. For the numerical estimate of the last line, the logarithmic factor was calculated using $B = 100$ Gauss and $T = 1$ eV. This logarithmic factor becomes larger at higher temperature, and smaller at higher magnetic fields. For example, at the maximum magnetic field in CamV, $B = 10,000$ Gauss, the numerical factor in Equation 2.20 decreases from 180 to 110.

The rate at which the parallel and perpendicular degrees of freedom come into equilibrium with each other is accurately described by the rate predicted from “classical” short-range collisions [43], as

$$
\nu_{\perp||} \equiv \frac{3}{2} \nu_{ee}.
$$

In the following chapters, I make no distinction between the perpendicular temperature, $T_{\perp}$, and the parallel temperature, $T_{||}$. In essence, I assume the degrees of freedom are in equilibrium.

In practice, I also ignore radial variations in the temperature, and describe the plasma with one value of $T$. In EV, $T$ is found by taking an error-weighted average of the measured (nearly uniform) values of $T_{\perp}(r)$ across the diameter of the plasma. In CamV, however, the temperature is only measured at $r = 0$, and here I use $T = T_{||}(0)$. I can experimentally verify that the variations are typically
small in EV, as shown in Figure 2.4, but I must assume that they are also small in CamV.

There are theoretical reasons why the temperature should be relatively uniform. The heat conduction theory of Dubin and O’Neil predicts relatively rapid transport of heat across magnetic field lines [21], and these predictions have been recently verified with experiments on pure-ion plasmas by Hollmann and Anderegg [39, 37].

2.6 Angular Momentum and Confinement

Pure-electron plasmas in Penning-Malmberg traps have exceptional confinement properties compared to neutral plasmas. Particles are energetically prevented from leaving along the trap axis simply by applying a sufficiently high voltage \( e|V_c| > e|\phi| + m v_r^2/2 \) to the end electrodes. The primary loss is in the radial direction, across magnetic field lines to the walls of the trap. However, this radial transport is constrained by the conservation of angular momentum, and confinement is guaranteed in an ideal cylindrically-symmetric trap. In reality, perfect cylindrical symmetry is never achieved, and confinement in a Penning-Malmberg trap is determined by the construction asymmetries which induce radial transport in the plasma (Chapter 3).

The total canonical angular momentum of a collection of charges in a Penning-Malmberg trap is

\[
\mathcal{L}_\theta = \sum_j \left[ m_j v_{\theta j} r_j - \frac{q_j B}{2c} (R_w^2 - r_j^2) \right],
\]

(2.22)

where \( m_j, v_{\theta j}, r_j, \) and \( q_j \) are respectively the mass, azimuthal velocity, radial position, and charge of the \( j \)th particle. The first term in the equation is the mechanical part of the angular momentum, and it is at least 100 times smaller
than the second term for the experiments in this thesis. By neglecting the first term and pulling $q_j = -e$ out of the sum, Equation 2.22 can be rewritten as

$$
\mathcal{L}_\theta = \frac{eB}{2c} N_{\text{ion}} (R_w^2 - \langle r^2 \rangle) ,
$$

(2.23)

where $\langle r^2 \rangle$ is the mean-square-radius of the plasma.

In this approximation, the canonical angular momentum, $\mathcal{L}_\theta$, is due solely to the “Poynting Vector” angular momentum, which comes from the crossed electric and magnetic fields. The $R_w^2$ term is associated with the electric field from the image charges in the wall and the $\langle r^2 \rangle$ term is due to the space charge electric field of the real charges in the trap. For convenience, the angular momentum is often written without the $R_w^2$ term; but as written in Equation 2.23, $\mathcal{L}_\theta$ approaches zero as the plasma is lost to the wall.

Conservation of angular momentum implies a constant mean-square-radius $(d\langle r^2 \rangle/dt = 0)$, and the plasma is restricted from expanding. This restriction does not hold for a neutral plasma, since the charge $q_j$ takes positive and negative values, and thus cannot be taken out of the sum in Equation 2.22. In a neutral plasma, ions and electrons can step out radially together while still conserving angular momentum, and $d\langle r^2 \rangle/dt \neq 0$.

In Chapter 3, I present measurements of the radial expansion caused by purposely breaking the azimuthal symmetry of the trap with the application of external fields. For this transport there is a net loss of angular momentum from the plasma. On the other hand, viscous transport, which is the subject of Chapter 4, is due to internal interactions, and the total angular momentum is conserved. In this case, the electrons rearrange themselves (some move in and some move out) without a change in $\langle r^2 \rangle$. 
2.7 Global Thermal Equilibrium

If the external torques are sufficiently weak (allowing for sufficiently long confinement times) a non-neutral plasma may relax to a state of global thermal equilibrium [55, 23]. The equilibrium state is characterized by a uniform temperature, $T_{eq}$, a uniform fluid rotation frequency, $\omega_{eq}$, and a nearly uniform density characterized by the central density $n_{eq}$. The three parameters $(T_{eq}, \omega_{eq}, n_{eq})$ are themselves uniquely determined by the total number of particles $N_{tot}$, total energy $H$, and total angular momentum $\mathcal{L}_\theta$ of the plasma [53, 59, 9].

Assuming a cylindrically symmetric trap with time independent confinement fields, both the angular momentum and the total energy are conserved. They thus enter the distribution function on equal footing [59, 9], as

$$f = n_\alpha \left( \frac{m}{2\pi T} \right) \exp \left\{ -\frac{1}{T_{eq}} \left[ h + \omega_{eq} \ell_\theta \right] \right\}.$$  \hfill (2.24)

Using the single-particle energy $h = m v^2 / 2 - e \phi$ and single-particle angular momentum $\ell_\theta \equiv m v_\theta r + e B r^2 / 2 c$, the velocity dependence can be seen to be a Maxwellian in a frame rotating with frequency $\omega_{eq}$.

By integrating over the velocity distribution, the radial density profile in global thermal equilibrium can be written as

$$n(r) = n_{eq} \exp \left\{ -\frac{1}{T_{eq}} \left[ \frac{m}{2} \left( \Omega_c \omega_{eq} - \omega_{eq}^2 \right) r^2 - e \phi(r) \right] \right\}. \hfill (2.25)$$

Solutions to this equation are described by a family of curves parameterized by the single parameter $\gamma$, defined as

$$\gamma \equiv \frac{m \left( \Omega_c \omega_{eq} - \omega_{eq}^2 \right)}{2\pi e^2 n_{eq}} - 1.$$  \hfill (2.26)

For typical values of $\gamma$, the density is approximately constant in radius for many plasma Debye lengths, $\lambda_D$, and then falls to zero on the scale of $\lambda_D$. For a pure-
electron plasma, the Debye length is defined as

$$\lambda_D \equiv \left( \frac{T}{4\pi e^2 n_p} \right)^{1/2} \tag{2.27}$$

$$\approx 0.235 \text{ cm} \left[ \frac{T}{1 \text{ eV}} \right]^{1/2} \left[ \frac{10^7 \text{ cm}^{-3}}{n_p} \right]^{1/2}$$

The observation [16] of the thermal equilibrium state is possible due to the separation of equilibrium and transport time scales. On the collisional time scale ($1/\nu_{ee} \sim 1 - 10 \text{ ms}$) thermal equilibrium is quickly established axially along each field line. Due to the azimuthal rotation and assumed $\theta$-symmetry, the plasma can be thought of as being composed of cylindrical shells. Each individual shell is itself in equilibrium, but different shells are not in equilibrium with each other. On a longer time scale, the plasma can evolve to a global thermal equilibrium state due to electron-electron interactions which cause the transport of heat and particles across the magnetic field.

For the plasmas studied, the time to come to global thermal equilibrium was found to be on the order of $\tau_{eq} \sim 1 - 10 \text{ sec}$. This equilibrium time, in turn, can be less than the expansion time due to inherent asymmetries, which ranges from $1/\nu_{(r^2)} \sim 10 - 1000 \text{ sec}$ (depending on the rigidity of the plasma).
<table>
<thead>
<tr>
<th>Param. [unit]</th>
<th>Symbol</th>
<th>“EV” apparatus</th>
<th>“CamV” apparatus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field [Gauss]</td>
<td>$B$</td>
<td>Range: 95 - 470, Typical: 190</td>
<td>Range: 100 - 10,000, Typical: 1,000</td>
</tr>
<tr>
<td>Temp. [eV]</td>
<td>$T$</td>
<td>0.5 - 4.0, Typical: 1.0</td>
<td>0.3 - 2.4, Typical: 1.0</td>
</tr>
<tr>
<td>Den. [cm$^{-3}$]</td>
<td>$\langle n_p \rangle$</td>
<td>(0.3 - 1) x 10$^5$, Typical: 5 x 10$^6$</td>
<td>(0.2 - 2) x 10$^4$, Typical: 10$^4$</td>
</tr>
<tr>
<td>Plas. L. [cm]</td>
<td>$\langle L_p \rangle$</td>
<td>4 - 31, Typical: 12</td>
<td>3.5 - 38, Typical: 10</td>
</tr>
<tr>
<td>Plas. R. cm</td>
<td>$R_p$</td>
<td>1.6 - 2.1, Typical: 2.0</td>
<td>1.2 - 1.8, Typical: 1.5</td>
</tr>
<tr>
<td>Debye cm</td>
<td>$\langle \lambda_D \rangle$</td>
<td>0.2 - 0.8, Typical: 0.33</td>
<td>0.1 - 0.5, Typical: 0.24</td>
</tr>
<tr>
<td>Cyclo. [cm]</td>
<td>$r_c$</td>
<td>0.005 - 0.05, Typical: 0.012</td>
<td>10$^{-1}$ - 0.03, Typical: 0.0024</td>
</tr>
<tr>
<td>Wall R. cm</td>
<td>$R_w$</td>
<td>- , Typical: 3.81</td>
<td>- , Typical: 3.5</td>
</tr>
<tr>
<td>Conf. L. [cm]</td>
<td>$L_c$</td>
<td>7.89 - 35.6, Typical: 15.78</td>
<td>7 - 42, Typical: 14</td>
</tr>
<tr>
<td>Total number</td>
<td>$N_{tot}$</td>
<td>(0.3 - 3) x 10$^9$, Typical: 10$^9$</td>
<td>(0.3 - 3) x 10$^9$, Typical: 10$^9$</td>
</tr>
<tr>
<td>Rigidity</td>
<td>$\langle R \rangle$</td>
<td>1 - 12, Typical: 5</td>
<td>1 - 1000, Typical: 15</td>
</tr>
<tr>
<td>$E \times B$ [1/sec]</td>
<td>$\langle f_E \rangle$</td>
<td>10$^5$ - 10$^6$, Typical: 4 x 10$^5$</td>
<td>10$^3$ - 10$^6$, Typical: 10$^5$</td>
</tr>
<tr>
<td>Boun. 1/sec</td>
<td>$f_b$</td>
<td>10$^6$ - 10$^7$, Typical: 2 x 10$^6$</td>
<td>10$^6$ - 10$^7$, Typical: 2 x 10$^6$</td>
</tr>
<tr>
<td>Cyc. 1/sec</td>
<td>$\Omega_c$</td>
<td>(2 - 9) x 10$^9$, Typical: 4 x 10$^9$</td>
<td>10$^9$ - 10$^{11}$, Typical: 2 x 10$^{10}$</td>
</tr>
<tr>
<td>Dioc. 1/sec</td>
<td>$f_{dioct}$</td>
<td>(0.3 - 4) x 10$^5$, Typical: 10$^5$</td>
<td>10$^3$ - 10$^5$, Typical: 2 x 10$^4$</td>
</tr>
<tr>
<td>Plas. 1/sec</td>
<td>$\omega_p$</td>
<td>(1 - 2) x 10$^8$, Typical: 10$^8$</td>
<td>(1 - 2) x 10$^8$, Typical: 2 x 10$^8$</td>
</tr>
<tr>
<td>Coll. 1/sec</td>
<td>$\langle \nu_{ee} \rangle$</td>
<td>15 - 200, Typical: 90</td>
<td>35 - 200, Typical: 180</td>
</tr>
<tr>
<td>Plasma Volt</td>
<td>$\phi_p$</td>
<td>-10 to -30, Typical: -20</td>
<td>-5 to -30, Typical: -20</td>
</tr>
<tr>
<td>Conf. Volt</td>
<td>$V_c$</td>
<td>-100, Typical: -100</td>
<td>-35 to -100, Typical: -100</td>
</tr>
</tbody>
</table>

**Table 2.2:** Typical, and range of, values for experimental plasma parameters.
Chapter 3

Asymmetry-Induced Transport

3.1 Overview

In this chapter, I present measurements of radial transport in a non-neutral plasma due to asymmetric external fields. In general, I create an asymmetry by applying voltages to sections of the trap wall near the end of the plasma. A schematic of the experiment is shown in Figure 3.1. The applied voltages break the azimuthal symmetry of the trap, and thus cause the radial transport of particles. I identify two transport regimes: “slightly-rigid” where the plasma rigidity $R \equiv \tau_b/f_E$ is moderate, $1 < R < 10$, and “highly-rigid” where the plasma rigidity is relatively large, $R > 20$. The measured transport rates follow simple, yet radically different, empirical scalings in the two different regimes.

In the slightly-rigid regime, the expansion rate decreases with rigidity as $\langle R \rangle^{-2}$ and is linearly dependent on the applied voltage, $V_a$. The unknown mechanism responsible for this so-called $V_a R^{-2}$ transport appears to “turn-off” for an average rigidity in the range $\langle R \rangle = 10 - 20$. The transport for a $V_a \sim 1$ Volt asymmetry drops precipitously, and a different mechanism dominates. In this highly-rigid regime, the transport is found to be independent of the the plasma rigidity and scales as $V_a^2$. The experimental results are summarized by Figure 3.45.
Figure 3.1: Schematic of the standard $m = 1$ applied asymmetry shown from the side and from the end. The asymmetry is applied over a fraction ($L_{pert}/L_p$) of the plasma.
In Section 3.2 I describe the asymmetry-induced transport experiments and define the expansion rate. Measurements presented in this section show that the plasma expands linearly with time at a rate that increases with applied voltage. The density of the plasma decreases in conjunction with the expansion.

The immediate response of the plasma to the applied asymmetry is shown in Section 3.3. The experimental asymmetries are decomposed into a Fourier series which vary as $e^{im\theta}$, and are labeled by their dominant azimuthal mode number, $m$. The plasma distorts to a bounce-averaged meta-equilibrium shape in response to the applied asymmetry. The negative electrons, counter-intuitively, move toward the negative applied voltages. In particular, I present measurements of the induced center-of-mass displacement, $D$, due to an $m = 1$ asymmetry. The plasma column is shown to respond to an $m = 1$ asymmetry by rigidly shifting off-axis, even when the plasma is only slightly-rigid.

In Section 3.4 I present radial transport measurements in the slightly-rigid regime ($1 < \mathcal{R} < 10$). The net expansion rate due to an applied asymmetry, $\Delta \nu_{(r^2)}$, is shown in Figure 3.17 to robustly follow a simple empirical formula; $\Delta \nu_{(r^2)} \propto V_a \langle \mathcal{R} \rangle^{-2}$: the expansion rate is linearly proportional to the applied voltage, $V_a$, and inversely proportional to the square of the average rigidity, $\langle \mathcal{R} \rangle$. The mechanism responsible for this transport is, as of yet, unknown, and I refer to this transport as “$V_a \mathcal{R}^{-2}$” transport. The linear dependence on $V_a$ is surprising, considering that most current theories predict either a $V_a^2$ or a $V_a^{1/2}$ dependence for asymmetry-induced transport. In Figure 3.15 the linear dependence on voltage is shown to hold over two orders of magnitude ($V_a = 0.1 - 10$ Volt) and for voltages as low (and even less than) the effective inherent asymmetry in the trap ($V_t \approx 0.4$ Volt).

Measurements of the global expansion rate and of the local flux, $\Gamma_x$, are also presented in Section 3.4 for different $m$ number perturbations. Surprisingly, these
measurements show that the global expansion rate is independent of the azimuthal mode number of the applied asymmetry. The local flux, on the other hand, does depend upon the \( m \) number of the asymmetry, and is found to vary with radial position as \( \Gamma_r \propto r^m \).

In Section 3.5 I describe the transition between the slightly-rigid and highly-rigid transport regimes. The observed scaling of the expansion rate with both voltage and rigidity change abruptly in the rigidity range of \( \mathcal{R} = 10 - 20 \). This transition indicates that the \( V_a \mathcal{R}^{-2} \) transport mechanism “turns off” as the rigidity is increased above \( \mathcal{R} = 10 \).

Measurements of the expansion rate in the highly-rigid regime (\( \mathcal{R} > 20 \)) are presented in Section 3.6. In this regime, the expansion rate is proportional to \( V_a^2 \) and independent of the rigidity. Both of these scalings are consistent with predictions of “rotational-pumping” theory [8]. However, measured scalings with plasma length and density indicate that an extension to standard rotational-pumping theory is needed for a better (and quantitative) comparison to these experiments.

In Section 3.7, I present measurements of induced transport due to: (1) asymmetries applied over the entire length of a short plasma (2) asymmetries applied near the middle of the plasma (3) two asymmetries applied to opposite ends of the plasma, and (4) time varying asymmetries of relatively low frequency. These experiments differ from the standard experiments, which consists of applying a static asymmetry to one end of the plasma. The most interesting result from these experiments is that the \( V_a \mathcal{R}^{-2} \) transport mechanism appears to “turn off” when the asymmetry is applied along the entire length of the plasma. In other words, the asymmetry must have an axial dependence for the \( V_a \mathcal{R}^{-2} \) mechanism to dominate.

In Section 3.8, I compare my results on transport due to an applied electric
asymmetry to other experimental measurements of asymmetry-induced transport. Experiments on plasmas in the slightly-rigid regime are shown to scale with rigidity as $R^{-2}$. This scaling holds for transport due to inherent asymmetries (on 5 different machines), applied magnetic asymmetries, and damping of the $m = 1$ diocotron mode on EV.

The experimental results are summarized in Section 3.9, and in Section 3.10 I suggest directions for future theoretical work. For plasmas in the slightly-rigid regime, the strong dependence of the measured transport on the plasma rigidity suggests that bounce-resonant particles play an important role. However, no current version of bounce-resonant theories predict the observed $V_a^{-1}$ scaling. In the highly rigid-regime, rotation-pumping is seen as the likely mechanism responsible for transport. The observed agreement and disagreement between the measurements and the theory are discussed in this section.

## 3.2 Description of Experiments

In this section, I describe the experimental approach used to obtain the measurements of radial transport presented in this chapter. I summarize this approach with three major points: (1) I measure the transport due to well controlled applied asymmetric voltages, $V_a$, as well as due to the inherent trap asymmetries, characterized by an “effective” voltage, $V_i$. (2) I quantify the transport with a global expansion rate, $\nu(r^2)$, which is proportional to the change in the total angular momentum of the plasma. (3) I calculate rates over relatively short times, $\Delta t$, where the plasma parameters are essentially fixed.

My standard $m = 1$ asymmetry consists of static voltages $+V_a$ and $-V_a$ applied to opposite azimuthal sectors at the end of the plasma, shown schematically in Figure 3.1. Here, $m$ refers to the dominant azimuthal mode number of the
asymmetry. The differences and similarities of the measured transport due to variations on this configuration, such as an $m = 2$ asymmetry, are described in this chapter.

I calculate transport rates and fluxes from the time evolution of the radial density profile, $n_c(r)$. The primary quantity I use to characterize asymmetry-induced transport is the rate of change of the mean square radius of the plasma,

$$\nu_r \equiv \frac{1}{\langle r^2 \rangle} \frac{d}{dt} \langle r^2 \rangle \propto -\frac{1}{L_\theta} \frac{dL_\theta}{dt},$$

(3.1)

where

$$\langle r^2 \rangle \equiv \frac{L_c}{N_{tot}} \int_0^{Rw} 2\pi r \, dr \, n_c(r) \, r^2.$$  (3.2)

The total number of electrons, $N_{tot}$, does not change within the 1% accuracy of the measurement. Therefore, $\nu_r$ is directly proportional to the rate of change of the angular momentum, $L_\theta$, and is a measure of the global effects of an asymmetry on the entire plasma.

The quantity $\langle r^2 \rangle$ can be broken up into two parts

$$\langle r^2 \rangle = \langle \rho^2 \rangle + D^2,$$  (3.3)

where $\langle \rho^2 \rangle$ is the mean-square-radius of the column calculated relative to the center-of-mass position; and $D$ is the displacement of the center-of-mass away from the trap axis, as defined in Figure 3.1. For the experiments presented in this thesis, $D^2$ is small compared to $\langle \rho^2 \rangle$. In addition, $D$ is constant while the plasma expands due to an applied asymmetry: $D$ only changes during the short turn-on (or turn-off) ramp of the applied voltage. Therefore, to a good approximation, I can take

$$\langle r^2 \rangle \approx \langle \rho^2 \rangle$$

and to an even better approximation, I can take

$$\frac{d\langle r^2 \rangle}{dt} \approx \frac{d\langle \rho^2 \rangle}{dt}.$$
There is one exception where the above approximations are not appropriate, and that is in the damping of the diocotron mode presented in Section 3.8.3.

As well as measuring the global transport rate, I also measure the local particle flux $\Gamma_z(r)$. This flux is calculated as described in Section 2.4 from the local change in the confinement density, $\Delta n_c(r)$, between two profiles with a difference in hold times of $\Delta t$. For fixed $N_{tot}$, the flux at the wall is zero ($\Gamma_z(R_w) = 0$), and Equations 2.4, 3.1, and 3.2 can be combined to show the relationship between the global expansion rate and the local measured quantities as

$$\nu(r^2) = \frac{2 \int_0^{R_w} dr \, r \, \Gamma_z(r) \, r}{\int_0^{R_w} dr \, n_c(r) \, r^2} = \frac{\int_0^{R_w} dr \, r \, (\Delta n_c(r)/\Delta t) \, r^2}{\int_0^{R_w} dr \, r \, n_c(r) \, r^2}. \quad (3.4)$$

I have measured the transport for a range in initial conditions characterized by the values of the axial magnetic field $B$, electron temperature $T$, average plasma length $\langle L_p \rangle$, average plasma density $\langle n_p \rangle$, and approximate radial size of the plasma $R_p$. The average rigidity $\langle R \rangle \equiv \langle B \rangle/\langle f_E \rangle$ is calculated from these parameters using the estimate given in Equation 2.19,

$$\langle R \rangle = 1.46 \left[ \frac{B}{100 \, \text{G}} \right] \left[ \frac{T}{1 \, \text{eV}} \right]^{1/2} \left[ \frac{10^7 \, \text{cm}^{-3}}{\langle n_p \rangle} \right] \left[ \frac{10 \, \text{cm}}{\langle L_p \rangle} \right]. \quad (3.5)$$

In general, the density and temperature profiles are initially made nearly uniform in radius using the technique of “tilt-smoothing”, which is just the rapid transport due to an applied asymmetry as further explained in this chapter.

After the plasma is created and manipulated (i.e. after tilt-smoothing), an applied voltage is ramped from 0 to $V_a$ in a time $t_{ramp} \sim 1 - 10 \, \text{ms}$; it is held on for a specified duration $t_{pert} \sim 10 \, \text{ms} - 1 \, \text{s}$; and then ramped off in a time $t_{ramp}$. (An example of the voltage time dependence is shown in Figure 3.3.) The ramp time is always much larger than the $m = 1$ diocotron period ($t_{ramp} \gg \tau_{dio}$) so that the
Figure 3.2: Effects of an applied $m = 1$ asymmetry on the measured density profile. EV density profiles, labeled by the asymmetry strength $V_a$, are shown after an $m = 1$ asymmetry has been applied for $t_{pert} = 0.09$ sec.

The applied asymmetry causes the plasma to expand radially and to decrease in density. In Figure 3.2, three EV density profiles with the same initial conditions and hold time ($t_{hold} = 0.4$ sec) show the effects of applying an external asymmetry. For two of the profiles, an $m = 1$ asymmetry of $V_a = 1$ V and $V_a = 2$ V was applied for $t_{pert} = 90$ ms. Comparing these profiles to the case with no applied asymmetry ($V_a = 0$), the density near the center of the plasma has decreased, and the density at the edge of the plasma has increased. The mean-square-radius of the plasma increased as $\langle r^2 \rangle \approx 2.0, 2.1, \text{ and } 2.2 \text{ cm}^2$ for $V_a = 0, 1, \text{ and } 2$ V, respectively.
Figure 3.3: Mean-square-radius $\langle \rho^2 \rangle \approx \langle r^2 \rangle$ versus time for four different asymmetry strengths. The plasma expands linearly with time due to an applied $m = 1$ asymmetry of strengths $V_a = 0, 1, 2, \text{ and } 3 \text{ V}$. The time dependence of the asymmetry is shown at the bottom of the plot.

Radial expansion also causes the plasma temperature (not shown) to increase due to Joule heating. For a typical experimental plasma, an increase in $\langle r^2 \rangle$ of about 0.1 cm$^2$ equates to a temperature increases of about 0.1 eV.

In Figure 3.3, a time series of $\langle \rho^2 \rangle$ values are shown for 4 different applied voltages $V_a$, including $V_a = 0$. This data is obtained from CamV images, where some images are taken with the asymmetry still “on”. The time dependence of the applied voltage is shown at the bottom of the figure. The vertical dotted lines mark the beginning and end of the asymmetry ramps, and the vertical dashed lines mark the point at which the asymmetry is at full strength.

While the asymmetry is on, the plasma is observed to expand linearly with
time at a rate that increases with the applied voltage. The expansion rate with \( V_a = 0 \) is a measure of the “background” transport due to inherent trap asymmetries.

Experimental values of the expansion rate are calculated in two different ways. For the EV data, I use the approximation

\[
\nu_{\langle r^2 \rangle} \approx \frac{1}{\langle r^2 \rangle} \frac{\Delta \langle r^2 \rangle}{\Delta t},
\]

(3.6)

where \( \Delta \langle r^2 \rangle \) is the change in \( \langle r^2 \rangle \) between two density profiles. The two profiles are from the same initial conditions but taken at two different hold times: immediately before \( t_1 \) and immediately after \( t_2 \) an asymmetry is applied \( (\Delta t \equiv t_2 - t_1 > t_{pert}) \). To calculate a rate, I divide by the average value of \( \langle r^2 \rangle \) for the two profiles.

Some CamV data is taken in the same manner as the EV data, by using density profiles before and after the asymmetry. However, for the majority of the CamV data, \( \nu_{\langle r^2 \rangle} \) is calculated from a time series of \( \langle \rho^2 \rangle \) values (as in Figure 3.3), where the plasma is dumped while the asymmetry is still on. For this data, I use a linear regression fit to find \( d\langle \rho^2 \rangle/dt \) and calculate the expansion rate from the approximation

\[
\nu_{\langle r^2 \rangle} \approx \nu_{\langle \rho^2 \rangle} \equiv \frac{1}{\langle r^2 \rangle} \frac{\delta \langle \rho^2 \rangle}{\delta t}.
\]

(3.7)

It appears that dumping with the asymmetry “on” does not adversely effect my density profile measurements. The total number of collected electrons is the same (within the 1% experimental accuracy) whether the asymmetry is on during the dump or not; in other words, the applied voltages do not trap particles during the dump. However, as an added precaution against spurious measurements, I generally apply the perturbation to the inject end of the plasma in CamV. Which, according to Figure 2.1, means that I apply the asymmetry to S4 and use H3 and L2 together as the inject gate.

For both EV and CamV measurements, a data set consists of the expansion
rate as function of $V_a$; that is, $\nu_{\langle r^2 \rangle}(V_a)$ for a particular set of plasma parameters. The net expansion from just the applied asymmetry, $\Delta \nu_{\langle r^2 \rangle}$, is found by subtracting the background expansion due to the inherent trap asymmetries, $\nu_{\text{bg}} \equiv \nu_{\langle r^2 \rangle}(0)$. For CamV data calculated from Equation 3.7 $\Delta \nu_{\langle r^2 \rangle}$ is defined as

$$\Delta \nu_{\langle r^2 \rangle}(V_a) \equiv \nu_{\langle r^2 \rangle}(V_a) - \nu_{\text{bg}}. \quad (3.8)$$

For EV data calculated from Equation 3.6, $\Delta \nu_{\langle r^2 \rangle}$ is determined in a similar manner except that the right side of the above equation must by multiplied by the factor $\Delta t/t_{\text{pert}}$. This factor takes into account the fact that the asymmetry is only applied for $t_{\text{pert}}$ and not the entire duration $\Delta t$.

The change of the central density, $n_c(0)$, is shown in Figure 3.4 as a function of time for the same CamV images as Figure 3.3. The rate at which the central density changes is defined as

$$\nu_{n_0} \equiv \frac{-1}{n_c(0)} \frac{d n_c(0)}{d t}. \quad (3.9)$$

For $V_a = 3$ Volts, the central density decreases linearly with time while the asymmetry is on, and one can see numerically that $\nu_{n_0} \approx \nu_{\langle r^2 \rangle}$. In this one specific case, the change in the central density is a good indication of the change in the angular momentum of the plasma. However, for $V_a = 0$ and $V_a = 1$ Volt, I find $\nu_{n_0} \approx 0$ which is not in agreement with the global expansion rate $\nu_{\langle r^2 \rangle}$.

In general, the transport rate near the center of the plasma is only a good indication of the global transport rate (i.e. $\nu_{n_0} \approx \nu_{\langle r^2 \rangle}$) when the perturbation is a sufficiently large $m = 1$ asymmetry. I will return to this point by analyzing the difference between local and global transport for different $m$ number asymmetries in Section 3.4.3.
Figure 3.4: Central density $n_c(0)$ versus time for an applied $m = 1$ asymmetry at four different voltages, $V_a$. The density decreases with time for the larger voltages ($V_a = 2 \text{ V}$ and $3 \text{ V}$), but is relatively fixed for the smaller voltages, ($V_a = 0$ and $1 \text{ V}$).

3.3 Bounced-Averaged Equilibrium

The entire axial extent of the column appears to distort in $(r, \theta)$ to an asymmetric shape in response to external voltages applied to only one end of the plasma. The applied asymmetry effectively distorts the equipotential contours in the plasma. For an $m = 1$ asymmetry, the distortion amounts to a shift in the center-of-mass position of the column by an amount $D$ (also written as $d \equiv D/R_w$) towards the negative applied voltage. This shift is well described by force-balance between the force due to the image charges, and the $z$-averaged force due to the asymmetry. Surprisingly, this description works for the entire experimental range.
of plasma rigidity from $1 < \langle R \rangle < 1000$.

### 3.3.1 Plasma Distortion

Asymmetric electron plasma equilibria had previously been studied by the non-neutral plasma group at the University of California, Berkeley. John Notte and collaborators created metastable asymmetric plasmas by applying an asymmetric perturbation over the entire length of a relatively short plasma [51]. They found that, counter-intuitively, the negative electrons move toward negative voltages on the wall. The shape of the asymmetric equilibria is well described by an equipotential model of Chu *et al* [4], which postulates that the applied field and the self-consistent space-charge potential sum to a constant along the radial edge of the plasma. The stability of these asymmetric equilibrium is thought to result from the column being in a maximum energy state [56].

Schematics of nominal $m = 1$, $m = 2$, and $m = 4$ asymmetries, used to obtain the data presented in this chapter, are shown in Figure 3.5. The schematics
show the voltages on a 4 sector ring (60° sections) with the frame of the ring grounded. The shaded regions in the figure represent qualitative pictures of the plasma response to these asymmetries.

The asymmetric vacuum potentials, \( \Phi_a(r, \theta) \), produced by voltages applied to the walls of the trap can be decomposed into 2-D Fourier components as

\[
\Phi_a(r, \theta) = \sum_m \Phi_{a,m} \left( \frac{r}{R_w} \right)^m e^{im\theta}.
\]

where \( \Phi_a(R_w, \theta) \) is determined by the applied voltages and the angular size of the sectors.

In general, the vacuum potential has an axial dependence that can be written as \( \Phi_{a,m}(z) = \sum \Phi_{a,m,k_z} e^{ik_z \pi z/L_p} \). In presenting the measurements, I am not concerned with axial mode numbers except in making the distinction between a \( k_z = 0 \) perturbation, where the asymmetry is applied over the entire plasma, and my typical \( k_z \neq 0 \) perturbation, where the asymmetry is only applied to a portion of the plasma.

The experimental asymmetries are not pure Fourier modes in \( \theta \). For example, the nominal \( m = 1 \) asymmetry has non-zero values of \( \Phi_{a,m} \) for all odd azimuthal mode numbers; the first three are

\[
\Phi_{a,1} = 0.637 V_a
\]

\[
\Phi_{a,3} = 0.424 V_a
\]

\[
\Phi_{a,5} = 0.127 V_a
\]

More complete fourier decompositions (up to mode number 20) are listed in Appendix A for typical experimental configurations.
Figure 3.6: CamV images of the plasma (a) with and (b) without an $m = 1$ applied asymmetry of $V_a = 10$ Volts. Without an applied asymmetry, the plasma is centered in the trap. With an $m = 1$ applied asymmetry, the plasma is shifted toward the negative applied voltage, denoted by the wall section labelled $-V_a$. The small white crosses mark the center-of-mass and the large white crosses mark the center of the cylinder.

3.3.2 Displacement due to an $m = 1$ Asymmetry

For an $m = 1$ asymmetry, the distortion of the plasma amounts to a displacement of the center-of-mass off axis by an amount $D$. Examples of CamV images of the dumped plasma taken with and without an applied $m = 1$ asymmetry of $V_a = 10$ Volts are shown in Figure 3.6. Without the asymmetry, the plasma is centered in the trap ($D \approx 0$), but with the asymmetry, the plasma is shifted toward the negative applied voltage an amount $D = 0.16$ cm.
Figure 3.7: Displacement, $d$, and angular position, $\theta_{\text{com}}$, of the center-of-mass, versus time for an $m = 1$ asymmetry. The position of the plasma is fixed throughout the duration of the perturbation. The horizontal dashed line is the angular position of the negative applied voltage.

In Figure 3.7, the center-of-mass displacement scaled to the wall radius,

$$d \equiv D/R_w,$$

is shown as a function of time for four different applied voltages. The angular position of the displacement, $\theta_{\text{com}}$, is shown in the top portion of the figure. The data in this figure is calculated from the same CamV images used to calculate the $\langle \rho^2 \rangle$ and $n_c(0)$ time series data shown in Figures 3.3 and 3.4.

For an $m = 1$ asymmetry, the center of mass moves off axis while the perturbation is ramped on. The column remains fixed at a new meta-equilibrium
position until the perturbation is ramped off, at which point it moves back on axis. In other words, $D$ and $\theta_{\text{car}}$ are constant while the plasma expands due to the asymmetry. The offset column is static in the lab frame, in contrast to a dynamical $m = 1$ diocotron mode, where the plasma orbits around the trap axis. In the case of a diocotron mode, the unbalanced image force causes the whole plasma column to drift around the center of the trap. However, for an applied asymmetry, the drift from the image charge is effectively cancelled by a drift in the opposite direction, which is induced by the force from the asymmetry.

**Shifted (not Tilted nor Kinked) Column**

In the experimental work by Notte et al, the asymmetry was applied over the entire axial extent of a short plasma. However, for most of the data presented in this chapter, the asymmetry is applied only over a fraction of the plasma, as shown in Figure 3.1. A question arises as to the axial dependence of the plasma distortion in this case: Does (a) the entire plasma shift off axis, (b) does it tilt, or (c) does it kink? Cartoon examples of the side view for a shifted, tilted, or kinked plasma are shown in Figure 3.8, along with an approximate $z$-integrated end view.

CamV images indicate that the entire plasma is shifted off axis rigidly, \emph{i.e.} it is neither tilted nor kinked. In Figure 3.6 the plasma appears to remain circular as its displaced off axis, as would occur for a shifted column, but not for a tilted or kinked column. The main quantitative evidence in support of this “shifted-only” hypothesis is found by comparing the calculated quadrupole moment (or ellipticity), $Q_2$, of the dumped plasma to expected values of $Q_2$ for models of shifted, tilted, or kinked plasmas. Here, the quadrupole moment is defined from the center of mass as
Figure 3.8: Schematics of a shifted, tilted, or kinked column from a side view and from a dump view ($z$-integration). In all three cases, the center of mass is displaced toward the negative voltage. The dump view for the shifted column is circular, while the tilted and kinked columns are somewhat elliptical.
Table 3.9: Measured image dumped with an $m = 1$ asymmetry on a slightly-rigid plasma ($\langle R \rangle \approx 5$) as compared to expected (i.e. modeled) images for shifted, tilted, or kinked columns. Calculated values of the quadrupole moments $Q_2$ for each image are listed. All of the images have the same center-of-mass displacement of $d = 0.047$, but only the measured and shifted images are roughly circular ($Q_2 \approx 0$). The difference in $\theta_{com}$ between the measured and expected images is irrelevant.

$$Q_2 = \frac{\int d\rho \rho^2 \int d\psi \cos(2\psi) n(\rho, \psi)}{\int d\rho \rho^2 \int d\psi n(\rho, \psi)},$$

(3.12)

where $\psi$ is the angular position of a cylindrical coordinate system centered on the center-of-mass of the column.

In Figure 3.9, I display the measured dump image image shown in Fig-
ure 3.6(a) along with expected images for a shifted, tilted, or kinked electron column. The expected images were calculated by distorting an initial profile: the profile was taken from a fit to the centered image of Figure 3.6(b). The imaginary electron column was shifted, tilted, or kinked by an amount such that the resulting center-of-mass displacement matched the measured value of $d = 0.047$. The difference in the angular position of the center-of-mass, $\theta_{\text{com}}$, between the measured and expected profiles in Figure 3.9 is irrelevant. The difference is simply an artifact of the program used to generate the expected images.

By comparing the calculated quadrupole moments, $Q_2$, the shifted column is shown to be a much better description of the measured image than either a tilted or a kinked column. The measured image is roughly circular with $Q_2 = 0.001$, consistent with $Q_2 = 0$ for the shifted column. On the other hand, a tilted or kinked plasma is expected to appear somewhat elliptical when dumped, with expected values of $Q_2$ that are roughly 20 and 80 times larger than the measured value.

The calculated rigidity for the plasma of the measured images in Figures 3.6(a) and 3.9 is only $\langle R \rangle = 5$. Surprisingly, the whole plasma is found to rigidly shift off axis even when it is only slightly rigid.

**FORCE BALANCE**

As the plasma moves off axis, image charges are induced in the wall of the trap. The image charges cause an electrostatic force in opposition to the force from the applied asymmetry. I find that the magnitude and parameter scaling of the center-of-mass displacement, $d$, is well described by force balance between the electrostatic forces from the image-charge and the asymmetry.

For an electron column displaced off-axis an amount $D$, Fine made the observation that the image charges can be considered as a line of charge with
charge per unit length $eN_L$ at a distance $R_w^2/D$ from the center of the trap [31].

The electric field from the image charges, $E_{image}$ is related to the displacement $D$ as

$$E_{image} = \frac{2eN_L D}{R_w^2}. \quad (3.13)$$

I approximate the bounded-averaged field, $E_{a,1}$ due to an $m = 1$ applied asymmetry using the simple formula

$$E_{a,1} = \frac{1}{N_{q_d}} \int dr \int d\theta \int dz \, n(r, \theta, z) E^3_{a,1}(r, \theta, z) \approx \frac{\Phi_{a,1}}{R_w} \left( \frac{L_{pert}}{\langle L_p \rangle} \right), \quad (3.14)$$

where $L_{pert}$ is the length of the plasma directly beneath the sectors as shown in Figure 3.1. In practice, I compute $L_{pert}$ from the calculated value of $\langle L_p \rangle$ along with the location and dimensions of the sectored ring used for the asymmetry.

The factor $(L_{pert}/\langle L_p \rangle)$ is intended to represent the fraction of the plasma which is “feeling” the perturbation at any given time. By using the approximation of Equation 3.14, I am essentially ignoring the fringing fields of the applied wall voltage. A more accurate estimate of $E_{a,1}$ can be obtained by numerically evaluating the integral in Equation 3.14.

Force balance (i.e. $E_{image} = E_{a,1}$) leads to an approximate theoretical relationship between the displacement $d$ and the $m = 1$ asymmetry strength $\Phi_{a,1}$ as

$$d^{th} = \frac{\Phi_{a,1}}{2eN_L} \left( \frac{L_{pert}}{\langle L_p \rangle} \right). \quad (3.15)$$

A similar equation can be derived from the equipotential model of Chu et al [4].

The measured displacement increases linearly with asymmetry strength, in agreement with Equation 3.15. Deviations from linearity only begin to appear when the applied voltage is greater than the magnitude of the space-charge potential at
\[ \frac{d}{D/R_w} = 3.5 \text{ cm} \]

\[ \frac{d}{D/R_w} = 9.6 \text{ cm} \]

\[ \frac{d}{D/R_w} = 19.8 \text{ cm} \]

\[ \frac{d}{D/R_w} = 30.5 \text{ cm} \]

**Figure 3.10:** Center-of-mass displacement from an \( m = 1 \) asymmetry as a function of \( V_a \) for four different plasma lengths and two different magnetic fields. The dashed lines are drawn to show the approximate \( V_a^{-1} \) scaling. The displacement is larger for shorter plasmas, but does not exhibit a magnetic field dependence.

the center of the plasma, which is typically about 20 Volts.

Measured values of \( d \) are shown in Figure 3.10 as a function of \( V_a \) for four different plasma lengths and two different magnetic fields. In this figure, the displacement is shown to be linearly proportional to the applied asymmetry strength over a wide range in plasma conditions. In addition, \( d \) is also shown to decrease with length, and to be independent of magnetic field, in agreement with Equation 3.15.

Surprisingly, the displacement does not depend upon the rigidity of the plasma. The plasma appears to be displaced in the same bounced-averaged sense for the entire range of data from \( 1 < \langle R \rangle < 1000 \) as shown by Figure 3.11. Each
Figure 3.11: Scaled center-of-mass displacement from an \( m = 1 \) asymmetry as a function of rigidity. The theoretical prediction \( d^{th} \) adequately describes the measured value of \( d \) over the entire experimental range \( \langle R \rangle = 1 - 1000 \).

Each point on this plot represents the results of fitting a data set of \( d \) vs \( V_a \) to the equation

\[
d = C_d V_a.
\]  

The fit is then scaled by the simple theoretical prediction \( d^{th} \) given in Equation 3.15, and I use the relationship between \( \Phi_{a,1} \) and \( V_a \) given in Equation 3.11. The standard deviation for the fits are generally smaller than the symbol size, and the main source of scatter is due to the simplifying estimates used to obtain \( d^{th} \).

In Figure 3.11 the theoretical prediction for the center-of-mass displacement \( d^{th} \) is in good agreement with the measured values of \( d \). The simple approximation of Equation 3.15 is shown to be only a slight over-estimate when the asymmetry
is applied to either the end, or near the center of the plasma. For all but four
data points (as described below) the measured values are within the range \( d/d^\text{th} = 0.6 - 1.0 \).

The single data point in Figure 3.11 with \( d/d^\text{th} > 1 \) shows the limitations of
the simple formula. For this point, the calculated value of \( L_{\text{pert}} \) is only 0.15 cm, and
the theoretical estimate \( d^\text{th} \) is exceedingly low. This underestimation is presumably
due to the neglect of the fringing fields. For all the other “Applied to end” data
(\( L_{\text{pert}} = 0.85-3.30 \) cm) and for the “Applied near center” data (\( L_{\text{pert}} = 3.94 \) cm),
the neglect of the fringing fields is an acceptable approximation.

When the asymmetry is applied over nearly the entire plasma Equation 3.15
is an over-estimate. For this situation the plasma is confined only in the sectored
ring (S4) of CamV. This data is plotted as circles in both Figure 3.11 and Figure
3.10. In calculating \( d^\text{th} \) I used the estimate \( L_{\text{pert}} = 3.2 \) cm \( \approx L_{\text{plasma}} \).

The shift in the equilibrium caused by an applied \( m = 1 \) asymmetry is
similar, in principle, to a physical shift of the trap wall, as shown in Figure 3.12. By
applying an \( m = 1 \) asymmetry to two sections of the wall, I shifted the equilibrium position (and the equipotential contours of the plasma) in just that cylinder by an amount \( \Delta \approx R_w \Phi_{a,1}/2eN_L \). Alternatively, the same effect can be achieved by physically moving grounded sections by the same amount, \( \Delta \). A relationship between \( \Delta \) and an applied \( m = 1 \) asymmetry of strength \( V_a \) can be written using \( \Phi_{a,1} = 0.637 V_a \), as

\[
\Delta \approx 0.044 R_w \left[ \frac{V_a}{1 \text{ Volt}} \right] \left[ \frac{5 \times 10^7 \text{ cm}^{-1}}{N_L} \right].
\]  

(3.17)

With a typical line density of \( N_L = 5 \times 10^7 \text{ cm}^{-1} \) and \( R_w \approx 4 \text{ cm} \) for EV and CamV, a 1 Volt \( m = 1 \) asymmetry is found to be equivalent to an effective shift (or misalignment) of two opposite sectors by an amount \( \Delta \sim 1 \text{ mm} \).

Note: for a plasma which spans a sectored cylinder with an applied asymmetry (or physical shift) and also spans grounded cylinder(s) (as shown in Figure 3.1) then the center-of-mass of the plasma lies somewhere in between the respective equilibrium lines for the cylinders (force balance), \textit{i.e.} \( D < \Delta \).

### 3.4 The Slightly-Rigid Regime: \( 1 < \langle \mathcal{R} \rangle < 10 \)

Measurements of the asymmetry-induced expansion rate \( \nu_{(r^2)} \) for a plasma in the slightly-rigid regime (\( 1 < \langle \mathcal{R} \rangle < 10 \)) are well described by the simple formula: \( \Delta \nu_{(r^2)} = (\nu_{(r^2)} - \nu_{bg}) = 7 V_a \langle \mathcal{R} \rangle^{-2} \). This simple formula has two striking characteristics:

- \( \Delta \nu_{(r^2)} \propto V_a^1 \) for all applied voltages from \( V_a = 0.1 \) to 10 Volts.
- \( \Delta \nu_{(r^2)} \propto \langle \mathcal{R} \rangle^{-2} \) for \( 1 < \langle \mathcal{R} \rangle < 10 \).

The average plasma rigidity was varied from 1 to 10 by changing the plasma parameters over the ranges \( B = 100 - 500 \text{ Gauss} \), \( \langle L_p \rangle = 10 - 40 \text{ cm} \), \( \langle n_p \rangle = \)
Figure 3.13: Expansion rate versus applied voltage for (a) 3 different magnetic fields and (b) 3 different temperatures. Dashed lines indicate that \( \nu_{(r^2)} \) increases approximately linearly with \( V_a \) from the background rate at \( V_a = 0 \).
Figure 3.14: Expansion rate versus applied voltage for (a) 3 different lengths and (b) 2 different densities. Dashed lines indicate that $\nu_{d}\nu$ increases approximately linear with $V_a$ from the background rate at $V_a = 0$, except for the data at $\langle L_p \rangle = 9.3$ cm, which has a rigidity of $\langle R \rangle \approx 12$. 
0.3 – 1.0 \times 10^7 \text{ cm}^{-3}, \text{ and } T = 1 – 4 \text{ eV}. \text{ For all the data in this section, the plasma has an effective radial size of } R_p/R_w = 0.4 – 0.6.

For each set of initial conditions, I measure the expansion rate \( \nu_{Q_x} \) over a range in applied voltage, \( V_a \). Sample EV data sets of \( \nu_{Q_x} \) as a function of \( V_a \) are shown in Figures 3.13 and 3.14. For this data, an \( m = 1 \) asymmetry was applied to the end of the plasma for \( t_{\text{pert}} = 90 \text{ ms} \). The expansion rate was calculated between density profiles taken immediately before and immediately after the asymmetry was applied, with a difference in hold time of \( \Delta t = 100 \text{ ms} \).

For each of the four graphs of Figures 3.13 and 3.14, the different data sets are labeled by one of the basic plasma parameters (the other parameters are kept relatively fixed). For all data sets, the plasma exhibits a “background” expansion rate, \( \nu_{\text{bg}} \), at \( V_a = 0 \), and \( \nu_{Q_x} \) increases above this background level as \( V_a \) is increased. The dashed lines show that the increase is approximately linear with voltage for all the data sets, except for “9.3 cm” in Figure 3.14(a) where \( \langle \mathcal{R} \rangle > 10 \).

### 3.4.1 Dependence on Applied Voltage: \( \Delta \nu_{(r^2)} \propto V_a^1 \)

For slightly-rigid plasmas (1 < \( \langle \mathcal{R} \rangle < 10 \)) the expansion rate increases approximately linearly with the strength of the asymmetry. This linear dependence is surprisingly inconsistent with predictions from standard theories, which vary either as \( V_a^2 \) or \( V_a^{1/2} \).

An example of the robustness of the linear dependence is shown in Figure 3.15, where \( \Delta \nu_{(r^2)} \propto V_a^1 \) over the range \( V_a = 0.1 – 10 \text{ Volts} \). To obtain \( \Delta \nu_{(r^2)} \) for this EV data, I subtract off the background expansion rate \( \nu_{\text{bg}} \) and scale by the relative duration \( t_{\text{pert}}/\Delta t \), as described in Section 3.2.

I find that the \( V_a^1 \) scaling holds even for relatively small applied voltages that are on the order of (or less than) the effective strength of the inherent trap
Figure 3.15: Linear increase of $\Delta \nu_{\phi, \gamma}$ with $V_a$ for a slightly-rigid plasma. The measured increase in the expansion rate for a plasma with follows the proportionality $\Delta \nu_{\phi, \gamma} \propto V_a^1$ (and not $V_a^2$ nor $V_a^{1/2}$) over two orders of magnitude. The data for which the asymmetry is applied for a longer duration have a larger average rigidity.

asymmetry, $V_t$. In Section 3.8.1, the effective trap asymmetry in EV (with the sectored ring) is estimated to be $V_t = 0.4$ Volt. In Figure 3.15, the linear dependence on voltage is shown to hold for values of $V_a < V_t$ and for $\Delta \nu_{\phi, \gamma} < \nu_{bg}$, where $\nu_{bg} = 0.42$ sec$^{-1}$.

In Figure 3.15, $\Delta \nu_{\phi, \gamma}$ is slightly smaller when the asymmetry is applied for a longer duration. This dependence on $t_{pert}$ is due to a dependence on plasma parameters. The longer an asymmetry is applied, the lower the plasma density and the higher the temperature. Both higher temperature and lower density lead to a decrease in the expansion rate as shown in Figures 3.13(b) and 3.14(b).
3.4.2 Dependence on Rigidity: $\Delta \nu_{(r^2)} \propto \langle R \rangle^{-2}$

The expansion rate decreases with the average rigidity of the plasma approximately as, $\Delta \nu_{(r^2)} \propto \langle R \rangle^{-2} \propto \langle L_p \rangle^2 \langle n_p \rangle^2 B^{-2} T^{-1}$. In other words, asymmetry-induced transport is weaker for more rigid plasmas. This observed dependence on rigidity is a strong indication that “bounce-resonant” particles significantly contribute to the observed transport.

Qualitative examples of the dependence on the individual plasma parameters ($B$, $\langle L_p \rangle$, $T$, and $\langle n_p \rangle$) can be seen in Figures 3.13 and 3.14. In these figures, the expansion rate is shown to increase with density $\langle n_p \rangle$ and length $\langle L_p \rangle$, and decrease with magnetic field $B$ and temperature $T$.

The decrease in transport with magnetic field is not too surprising, since one would expect it to be harder for charged particles to cross stronger field lines. The observed increase in transport with an increase in density and a decrease in temperature, could be explained by the fact that in both cases the collisionality of the plasma increases.

The length dependence is the most surprising, a priori. Intuitively, one might expect the asymmetry to become less significant and the transport to decrease as the length is increased (just as $d$ is found to decrease with increasing length as described in Section 3.3). As the plasma length is increased, the portion of the plasma “feeling” the asymmetry ($L_{perp}/\langle L_p \rangle$) decreases. The fact that the expansion rate is found to increase with length is one of the main motivations in expressing the parameter dependence in terms of the plasma rigidity.

A $\Delta \nu_{(r^2)} \propto \langle R \rangle^{-2}$ rigidity dependence in the slightly-rigid regime is shown in Figure 3.16. The $V_a^1$ voltage dependence is removed in this figure, by plotting the linear fit coefficient $C_1$, where $C_1$ is found by fitting each $\Delta \nu_{(r^2)}$ versus $V_a$ data.
Figure 3.16: Expansion rate fit coefficient, $C_1$, versus plasma rigidity in the regime $1 < \langle \mathcal{R} \rangle < 10$, where $C_1 \equiv \Delta \nu_{(r^2)}/V_a$ is found from fits to each data set of $\Delta \nu_{(r^2)}$ versus $V_a$. In the slightly-rigid regime the transport decreases as $\langle \mathcal{R} \rangle^{-2}$.

set to a line as

$$\Delta \nu_{(r^2)} = C_1 V_a.$$  \hspace{1cm} (3.18)

For example, the data shown in Figure 3.15 has an average rigidity of $\langle \mathcal{R} \rangle = 2.5$, and is fit by the line $\Delta \nu_{(r^2)} = 1.2 V_a$, \textit{i.e.} $C_1 = 1.2$.

The full range of the data is shown in Figure 3.17. Here, I plot each measured value of the net expansion rate \textit{i.e.} no fit, having $1 < \langle \mathcal{R} \rangle < 10$, versus the quantity $V_a \langle \mathcal{R} \rangle^{-2}$. All of the data is shown to be well described by the simple empirical formula

$$\Delta \nu_{(r^2)} = 7 \text{sec}^{-1} \left[ \frac{V_a}{\text{Volt}} \right] \langle \mathcal{R} \rangle^{-2}$$  \hspace{1cm} (3.19)
\[ \Delta \nu_{l^2} = 7 V_a (\langle R \rangle)^{-2} \]

This one formula accurately describes asymmetry-induced transport data on two different machines, over more than two orders of magnitude in the expansion rate.

In general, the expansion rate from an applied asymmetry is measured to be about 30\% less on CamV than on EV. I believe this difference is caused by the fact that \( L_{\text{pert}} \) is about 40\% shorter in the CamV experiments due to shorter electrodes. This difference suggests another (as yet untested) dependence to Equation 3.19, of the form \( \Delta \nu_{l^2} \propto L_{\text{pert}} \).

The simple formula given by Equation 3.19 is powerful, in that, with an asymmetry of known strength, one can get a general prediction of the transport
Figure 3.18: Expansion rate versus $V_a \langle R \rangle^{-2}$ for $m = 2$, $m = 4$, and one-sector applied asymmetries. The measured global expansion rate $\Delta \nu \langle r^2 \rangle$ for these $m \neq 1$ asymmetries follow the same simple formula used to describe $\Delta \nu \langle r^2 \rangle$ for an $m = 1$ asymmetry.

from one single dimensionless parameter, the rigidity. Conversely, by measuring the expansion rate for a plasma of a given rigidity one can calculate the approximate strength of an unknown asymmetry, e.g. inherent trap asymmetries (Section 3.8.1).

3.4.3 Dependence on $m$ Number.

Surprisingly, the global expansion rate $\nu \langle r^2 \rangle$ does not depend on whether the voltage $V_a$ has a predominantly $m = 1$, $m = 2$, or $m = 4$ azimuthal dependence. However, the radial dependence of local transport quantities, such as the experimental flux $\Gamma_x(r)$, does depend upon the $m$ number of the applied asymmetry.
Asymmetry-induced transport from an $m = 2$, $m = 4$, and a one-sector asymmetry are shown in Figure 3.18, where a one-sector asymmetry has substantial $m = 0, 1, 2, \& 3$ components as shown in Appendix A (note: an $m = 0$ perturbation does not cause radial transport as expected). The measured expansion rates $\Delta \nu(r, z)$ for the three different types of asymmetries are all within a factor of 2 of the simple equation $\Delta \nu_{(r, z)} = 7V_{a} \langle R \rangle^{-2}$, which was shown to describe transport for an $m = 1$ asymmetry in Figure 3.17.

In contrast, the local change in the measured density $\Delta n_{e}(r)/\Delta t$ (i.e. local flux) are markedly different for $m = 1$, $m = 2$, and $m = 4$ applied asymmetries. The upper curves of Figures 3.19 and 3.20 show $\Delta n_{e}/\Delta t$ versus $r$ for (a) inherent asymmetries ($V_{a} = 0$); (b) an applied $m = 1$ asymmetry; (c) an applied $m = 2$ asymmetry; and (d) an applied $m = 4$ asymmetry. In the lower portion of each of the four figures, density profiles, immediately before (dotted) and after (solid) the asymmetry is applied are shown.

A nominal $m = 1$ asymmetry is shown to cause a near uniform decrease in the plasma density across the interior of the plasma (this is evidence that the fields are not shielded), with a subsequent increase in density at the edge of the plasma. On the other hand, an $m = 2$ asymmetry causes little transport near the center of the plasma, and an $m = 4$ asymmetry only causes significant density changes at the very edge of the plasma.

Figure 3.21 shows that the radial fluxes $\Gamma_{x}(r)$ for the different $m$ number perturbations (calculated from the $\Delta n_{e}(r)/\Delta t$ data in Figures 3.19 and 3.20) vary approximately as

$$\Gamma_{x}(r) \propto r^{m}, \text{ which implies } \frac{\Delta n_{e}(r)}{\Delta t} \propto r^{m-1}$$

over the interior of the plasma. The dotted curve in the figure is the initial radial
Figure 3.19: Density profiles and $\Delta n_c(r)/\Delta t$ for transport due to (a) the inherent asymmetries ($V_a = 0$), which cause a small amount of transport at the edge of the plasma; and (b) an applied $m = 1$ asymmetry ($V_a = 1$ Volt), which causes a near uniform decrease in density across the bulk of the plasma.
Figure 3.20: Density profiles and $\Delta n_c(r)/\Delta t$ for transport due to an applied (c) $m = 2$ and (d) $m = 4$ asymmetries of $V_a = 1$ Volt. Neither the $m = 2$ nor the $m = 4$ applied asymmetry cause significant changes in the central density, $n_c(0)$, over the short duration of the experiment ($\Delta t = 0.033$ sec).
**Figure 3.21:** Measured flux versus radius for 3 different $m$ number asymmetries. The measured fluxes vary as $r^m$ near the interior of the plasma. The flux due to the inherent asymmetries is also shown, and the dotted curve represents the initial radial density profile, $n_c(r)$.

density profile. As expected, the calculated flux goes to zero as the density goes to zero.

At the peak in the measured flux, near the radial edge of the plasma, the fluxes for the different $m$ numbers are all nearly equal to each other. This leads to the observed near equality in the global expansion rates $\nu_{(r^2)}$, since this quantity is dominated by transport at the edge of the plasma. Transport near the center of the plasma contributes only weakly to $\nu_{(r^2)}$; (see Equation 3.4).
3.4.4 Local Model

Presumably, the observed $r^m$ dependence in the flux occurs due to the fall-off in the asymmetry fields away from the trap wall. In Equation 3.10 the vacuum fields are shown to vary as $\Phi_a(r) \propto (r/R_w)^m$. I suggest here a local model of the flux for a given $m$ number perturbation, $\Gamma_m$, which depends upon the vacuum fields, \textit{i.e.} no shielding, as

$$
\Gamma_m(r) = \alpha n_e(r) \Phi_{a,m} \left( \frac{r}{R_w} \right)^m \left( \frac{m f_E(r)}{f_b(r)} \right)^2 ,
$$

(3.21)

where $\alpha$ is a constant of proportionality. By integrating this formula according to Equation 3.4, the model can be shown to be consistent with the empirical expansion rate (Equation 3.19), and alpha can be approximated as

$$
\alpha \approx 5 \times 10^3 \left[ \frac{\text{cm}}{\text{sec Statvolt}} \right] .
$$

(3.22)

Except for the voltage dependence, this empirical model of the flux is quite similar to a simple estimate of the flux due to resonant particles transport, for a plasma in the so called banana-regime [29]. I will return to this comparison in the final section of this chapter.

The near equality in the flux at the edge of the plasma can be explained with Equation 3.21 by summing over the different Fourier terms (up to $m = 10$, see Appendix A). The total predicted flux, \textit{i.e.} $\Gamma_{tot} = \sum_m \Gamma_m$, at the radial edge of the plasma, $r/R_w = 0.5$, is found to be the same (within 10%) for the experimental $m = 1$, $m = 2$, $m = 4$, and one-sector asymmetries. Note, that for the $m = 1$ asymmetry the predicted flux at the edge of the plasma has a considerable contribution from the higher order $m = 3$ and $m = 5$ components.

The local flux model given by Equation 3.21 predicts radial variations in $\Gamma_m(r)$ due to variations in the local rigidity of the plasma $\mathcal{R}(r) \equiv \mathcal{f}_b(r)/f_E(r)$. I
Figure 3.22: Change in density caused by an $m = 1$ applied asymmetry on a peaked density profile. The local change in the density $\Delta n_c(r)/\Delta t$ is larger for the density peak, and the profile is effectively “smoothed”.

find that these predictions agree qualitatively with measurements of “tilt transport”, which is the flattening of a peaked density profile by the application of an asymmetry with an $m = 1$ azimuthal dependence. Fine reported obtaining flat density profiles with a magnetic “tilt” [31], which I show in Section 3.8.2 to be nearly identical to an applied $m = 1$ electric asymmetry.

In Figure 3.22, I show the “smoothing” effects of an $m = 1$ electric asymmetry on a peaked profile. The local change in density $\Delta n_c(r)/\Delta t$ is larger for the density peak, which corresponds to a minimum in the local rigidity. For this plasma, the local rigidity varies from $\mathcal{R} = 1.1$ at the center of the plasma to $\mathcal{R} = 2.4$ at the edge.
The global expansion rates for peaked plasmas are just as well described by
the simple equation \( \Delta \nu(r, z) = 7 V_a \langle R \rangle^{-2} \) as are plasmas with flat density profiles. For example, the profile in Figure 3.22 has an average rigidity of \( \langle R \rangle = 1.8 \) and a measured expansion rate of \( \Delta \nu(r, z) = 1.8 \text{ sec}^{-1} \), which is within 20\% of the predicted value of 2.2 sec\(^{-1}\) as calculated from Equation 3.19.

### 3.4.5 Rate of Change of Central Density: \( \nu_{n_0} \)

In this section, I present measurements of the rate of change of the central
density, \( \nu_{n_0} \) (Equation 3.9). These measurements indicate that the applied field
is only partially shielded from the center of the plasma for relatively small voltages. At higher applied fields, the asymmetry appears to saturate across the entire plasma; this saturation occurs at surprisingly low voltages.

In Figure 3.23, \( \nu_{n_0} \) is plotted versus the strength of the \( m = 1 \) Fourier component at the trap wall, \( \Phi_{a,1} \), for a nominal \( m = 1 \) asymmetry (\( \Phi_{a,1} = 0.637 V_a \)) and a one-sector asymmetry (\( \Phi_{a,1} = 0.318 V_a \)). For the \( m = 1 \) asymmetry I present data for two different cases: a standard asymmetry applied to the end of the plasma, and an asymmetry applied near the middle of the column (see Section 3.7.2). For one-sector asymmetries, I present data for both an applied positive or negative voltage; there is no substantial difference between the induced transport for the two cases. The background (\( V_a = 0 \)) rate of change of the central density is zero to within the accuracy of the measurement (i.e. \( \Delta \nu_{n_0} \approx \nu_{n_0} \)).

In Figure 3.23, the expansion rate at the center of the plasma is shown to be
linearly proportional to \( \Phi_{a,1} \) for \( \Phi_{a,1} > 0.5 \) Volt (corresponding to \( V_a \gtrsim 1 \) Volt for a nominal \( m = 1 \) asymmetry). For smaller asymmetry strengths (\( \Phi_{a,1} \lesssim 0.5 \) Volt), the measured transport drops below this \( \nu_{n_0} \propto \Phi_{a,1}^1 \) scaling line, and more closely follows the scaling \( \nu_{n_0} \propto \Phi_{a,1}^2 \). (I remind the reader that the global expansion rate
\[ \langle R \rangle = 2.6 \]

![Graph](image)

**Figure 3.23:** Rate of change of the central density, \( \nu_{n_0} \), versus the \( m = 1 \) component of an applied asymmetry at the trap wall, \( \Phi_{a,1} \).

\( \Delta \nu_{(r^2)} \), and the rate of change of the density at the plasma edge, is found to be linearly proportional to \( V_a \) even for \( 0.1 < V_a < 1 \), see Figure 3.15.)

The relative magnitude of \( \nu_{n_0} \) scaled by the global expansion rate \( \Delta \nu_{(r^2)} \) is plotted in Figure 3.24 as a function of the applied asymmetry voltage at the trap wall, \( V_a \). For the smaller voltages, the asymmetry appears to be shielded somewhat from the center of the plasma, since \( \nu_{n_0} < \nu_{(r^2)} \). As the applied voltage increases, the shielding becomes weaker until it appears to saturate at about \( \nu_{n_0} \approx \nu_{(r^2)} \). This saturation occurs at approximately the same point where the transition from a \( \nu_{n_0} \propto \Phi_{a,1}^1 \) scaling to a \( \nu_{n_0} \propto \Phi_{a,1}^2 \) scaling occurs as shown in Figure 3.23. Note that the strength of the vacuum fields is much lower near the center of the plasma \((r/R_w < 0.05)\) than the quantities \( V_a \) and \( \Phi_{a,1} \) which are both a measure of the
**Figure 3.24:** Ratio of the rate of change of the central density to the global expansion rate, $\nu_{n_0}/\Delta \nu_{(g^2)}$, versus voltage applied at the trap wall, $V_a$, for a nominal $m = 1$ asymmetry and a *one-sector* asymmetry. The ratio increases with $V_a$ up until $\nu_{n_0}/\Delta \nu_{(g^2)} \approx 1$.

asymmetry strength at the trap wall.

The global expansion rate, which is dominated by the transport at the edge of the plasma, does not exhibit a similar transition from $V_a^2$ to $V_a^1$. Presumably the plasma response is saturated at the edge of the plasma for even the smallest applied asymmetries ($V_a = 0.1$ Volt), as well as for the inherent asymmetry ($V_i \approx 0.4$ Volt).

The $m = 1$ component ($\Phi_{a,1}$) for both nominal $m = 2$ and $m = 4$ applied asymmetries is zero. Therefore, according to the local empirical flux model (see Equations 3.20 and 3.21) there should be no change in the central density regardless of the asymmetry strength (*i.e.* $\nu_{n_0} = 0$ for all $V_a$). However, for relatively
large values of $V_a$ and/or for relatively long asymmetry durations, $t_{pert}$, the central density is seen to decrease for $m \neq 1$ asymmetries.

I believe that the eventual decrease in the central density with an $m = 2$ or $m = 4$ asymmetry is due to viscous transport. In Figure 3.20(c), an $m = 2$ asymmetry is shown to cause an initially flat density profile to become peaked at the center. A peaked density profile equates to a peak in the rotation frequency. Viscous transport, presented in the following chapter, acts to eliminate shears in the plasma and smooths out such density peaks.

Viscous transport is believed to be due to internal (like-particle) interactions, which conserve the total angular momentum. Therefore, the global quantity $\nu_{g2}$ (which is proportional to the change in the angular momentum) is not affected by the presence of viscous transport, and is an unambiguous measure of asymmetry-induced transport. Local transport quantities such as $\nu_{n0}$ and $\Gamma_x(r)$, on the other hand, can represent a combination of both asymmetry and viscous transport. If an experimentalist wishes to measure the local effects of just one of these types of transport, s/he must take care to reduce the transport from the other type. For instance, the local flux measurements of Figure 3.21 were performed on flat density profiles where the viscous transport is minimized.

3.5 Transition Between Regimes: $\langle R \rangle = 10 - 20$

The parameter scalings of asymmetry-induced transport change dramatically as the rigidity increases from $\langle R \rangle \approx 10$ to $\langle R \rangle \approx 20$. In the “highly-rigid” regime of $\langle R \rangle > 20$, the transport process, described in the previous section, appears to “turn off”, and a different transport process dominates. (In Section 3.9 I discuss this turn off in relation to a bounce adiabatic invariant.) The two most obvious differences between the measured transport in the two regimes are in the
scaling of the expansion rate with asymmetry strength and with rigidity.

In Figure 3.25, the measured expansion rate is plotted as a function of applied voltage for an \( m = 1 \) asymmetry at 6 different values of the rigidity, spanning the range \( \langle \mathcal{R} \rangle = 1.5 - 130 \). For \( \langle \mathcal{R} \rangle < 10 \), the plasma is slightly-rigid and the expansion rate decreases with increasing rigidity and depends linearly on the applied voltage. For \( \langle \mathcal{R} \rangle \gtrsim 20 \), the expansion rate becomes independent of the rigidity and varies as \( V_\alpha^2 \).

The transition from a \( V_\alpha^3 \) to a \( V_\alpha^2 \) dependence has been observed with decreases in length and density, as well as with an increase in magnetic field. All of the observed transitions occur in the range \( \langle \mathcal{R} \rangle = 10 - 20 \). For example, in Figure 3.14 the expansion rate for a length of 9.3 cm at \( B = 380 \) Gauss exhibits more of a \( V_\alpha^2 \) dependence than a \( V_\alpha^3 \) dependence; for this data I calculate \( \langle \mathcal{R} \rangle = 12 \).

### 3.6 Transport in the Highly-Rigid Regime

In the highly-rigid regime (\( \langle \mathcal{R} \rangle > 20 \)) the measured expansion rate is described by the empirical formula:

\[
\Delta \nu_{(r)} = 5 \times 10^{-4} \text{sec}^{-1} \left( \frac{V_\alpha}{1 \text{ Volt}} \right)^2 \left[ \frac{1 \text{ eV}}{T} \right]^{\beta} \left[ \frac{10^7 \text{ cm}^{-3}}{\langle n_p \rangle} \right].
\]  

(3.23)

where \( \beta = 1/2 \) for \( T < 1 \) eV and \( \beta = 1 \) for \( T > 1 \) eV. This formula differs from the corresponding formula for transport in the slightly-rigid regime in two main points:

- \( \Delta \nu_{(r)} \propto V_\alpha^2 \) for all applied voltages from \( V_\alpha = 0.06 \) to 40 Volts.

- \( \Delta \nu_{(r)} \) is independent of rigidity \( \langle \mathcal{R} \rangle \) and of magnetic field strength \( B \).

Both of these points agree with the transport being due to the “rotational-pumping” of parallel energy [6, 8].
\[
\langle n_p \rangle = 0.7 - 1.0 \times 10^7 \text{ cm}^{-3}
\]
\[
\langle L_p \rangle = 20 \text{ cm}
\]
\[
T = 1.3 - 2.0 \text{ eV}
\]

**Figure 3.25:** Expansion rate versus voltage at 6 different values of \(\langle R \rangle\), spanning the range \(\langle R \rangle = 1.5 - 130\). For \(\langle R \rangle < 10\): the expansion rate depends linearly on the applied voltage and decreases with \(\langle R \rangle\). For \(\langle R \rangle \gtrsim 20\): the expansion rate varies as \(\Delta \nu_{(g^2)} \propto V_a^2\) and is independent of \(\langle R \rangle\).

To obtain highly-rigid plasmas, I use the CamV apparatus at high magnetic field. The data I present in this section spans the range in rigidity \(\langle R \rangle = 18 - 290\) with \(B = 1 - 10\) kG, \(\langle L_p \rangle = 10 - 38\) cm, \(T = 0.3 - 2.0\) eV, and \(\langle n_p \rangle = 0.2 - 1.6 \times 10^7\) cm\(^{-3}\).
3.6.1 Dependence on Applied Voltage: $\Delta \nu_{\langle r^2 \rangle} \propto V_a^2$

For a plasma in the highly-rigid regime, I find that the measured expansion rate increases quadratically with the applied voltage. In Figure 3.26, the expansion rate is shown versus $V_a$ for plasmas with an average rigidity of (a)$\langle R \rangle = 23$ and (b)$\langle R \rangle = 60$. The measured expansion rates are shown to follow the proportionality $\Delta \nu_{\langle r^2 \rangle} \propto V_a^2$. In Figure 3.26(b) the scaling is shown to hold for nearly 3 decades in the expansion rate, $\nu_{\langle r^2 \rangle}$, and over the range in applied voltage from $1 \leq V_a \leq 40$ Volts. For the data in this figure (and for most of the data in this section) the background transport is zero ($\nu_{bg} \approx 0$) within the uncertainty of the measurement ($i.e.$ $\nu_{\langle r^2 \rangle} = \Delta \nu_{\langle r^2 \rangle}$); this is because the magnetic field is relatively large.

The mechanism responsible for transport in the slightly-rigid regime appears to “turn-off” for $\langle R \rangle > 20$. Strong evidence for this turn off is shown in Figure 3.26(a), where the measured rate is shown to be as much as a factor of 10 less than the simple formula $\Delta \nu_{\langle r^2 \rangle} = 7V_a \langle R \rangle^{-2}$ for slightly-rigid transport.

3.6.2 Independence of Rigidity: $\nu_{\langle r^2 \rangle} \propto \langle R \rangle^0$

The transport does not depend strongly upon the plasma rigidity for $\langle R \rangle \gtrsim 20$, as shown in Figure 3.27. The $V_a^2$ voltage dependence is removed by plotting the quadratic fit coefficient, $C_2$, where $C_2$ is found by fitting each $\Delta \nu_{\langle r^2 \rangle}$ versus $V_a$ data set to the equation

$$\Delta \nu_{\langle r^2 \rangle} = C_2 V_a^2.$$  \hfill (3.24)

For example, fits to the data in Figure 3.26 yield the values (a) $C_2 = 1.0 \times 10^{-3} \text{sec}^{-1} \text{Volt}^{-2}$ and (b) $C_2 = 5.4 \times 10^{-4} \text{sec}^{-1} \text{Volt}^{-2}$.

Both the quantity $C_2$ and the linear fit coefficient $C_1$ (defined in Equation 3.18) can be thought of as the expansion rate at $V_a = 1$ Volt. The solid line in Figure 3.27 is from $\Delta \nu_{\langle r^2 \rangle} = 7V_a \langle R \rangle^{-2}$ with $V_a = 1$ Volt, and shows (again)
Figure 3.26: Expansion rate versus applied voltage for highly-rigid plasmas, (a) \( \langle R \rangle = 23 \) (b) \( \langle R \rangle = 60 \). In the highly-rigid regime \( \langle R \rangle > 20 \), the measured transport follows the scaling \( \Delta v_{r_2} \propto V_a^2 \), and is not described by the simple formula \( \Delta v_{r_2} = 7 V_a \langle R \rangle^{-2} \) (solid curve).
that the parameter scaling used to describe the slightly-rigid data does not apply to data in the highly-rigid regime.

In Figure 3.27, the single diamond point is the quadratic fit coefficient for a data set with an applied $m = 2$ asymmetry. This is the only data set I took in the highly-rigid regime for an $m \neq 1$ asymmetry. For this one example, the expansion rate for an $m = 2$ asymmetry scales as $V_a^2$ with an overall magnitude that agrees with transport from an $m = 1$ asymmetry.
Figure 3.28: Expansion rate fit coefficient, $C_2$, versus magnetic field strength for $\langle R \rangle > 20$. The expansion rate is roughly independent of magnetic field.

3.6.3 Independence of Magnetic Field: $\Delta \nu_{\langle r^2 \rangle} \propto B^0$

The transport process which dominates in a highly-rigid plasma does not depend strongly upon the magnetic field. In Figure 3.25 of the previous section, the measured expansion rates are roughly equal for $B = 2$, 5, and 10 kGauss over a wide range in $V_a$. The fit coefficient, $C_2$, for these data sets along with fit coefficients for 3 other data sets at lower $\langle n_p \rangle$ are shown in Figure 3.28 to be roughly independent of magnetic field. The solid line shows a $B^{-2}$ scaling for comparison.

It is somewhat counter-intuitive for a process to give transport of particles across magnetic field lines, yet be independent of the magnetic field strength. However, $B$-independent transport was previously observed in pure-electron plasmas at
Figure 3.29: Expansion rate versus Temperature for $\langle R \rangle > 20$ for fixed asymmetry strength, $V_a = 10$ V. As $T$ increases $\Delta \nu_{\langle r^2 \rangle}$ decreases. The center-of-mass displacement $d$ is independent of $T$.

these relatively high magnetic fields in the “CV” (Cryogenic) apparatus. The expansion of the plasma in the CV experiments are accurately described by a theory of “rotational-pumping” of parallel energy [6, 8]; the original work on this subject is briefly discussed in Section 3.10.2. An appropriate application of rotational-pumping theory may also describe the highly-rigid transport measurements of this section, but this has not yet been completed.

3.6.4 Dependence on Temperature: $\Delta \nu_{\langle r^2 \rangle} \propto T^{-1}$ or $\propto T^{-1/2}$

The expansion rate is shown to decrease with increasing plasma temperature in Figure 3.29. For $1 \leq T \leq 2.0$ eV, $\Delta \nu_{\langle r^2 \rangle}$ approximately follows the scaling
\( \Delta \nu_{(r^2)} \propto T^{-1} \), and for \( 0.3 \leq T < 1 \) eV, \( \Delta \nu_{(r^2)} \propto T^{-1/2} \). The center-of-mass displacement \( d \) is also plotted, to show that it is independent of temperature, as expected from the theoretical formula for \( d^{th} \) (Equation 3.15).

The measured expansion rates plotted in Figure 3.29 are due to a 10 V \( m = 1 \) asymmetry applied for \( t_{pert} = 1.0 \) s. The magnetic field was \( B = 10 \) kG, and the temperature was varied by allowing the plasma to cyclotron radiate over a given duration before beginning the experiment. The longest delay between injection and onset of the experiment was 20 sec. The background transport due to inherent trap asymmetries is exceedingly low at this high magnetic field, meaning that the initial conditions (besides the temperature) do not change substantially while the plasma is cooling. For example, the average density only decreased by 2\% in 20 sec.

### 3.6.5 Dependence on Density: \( \Delta \nu_{(r^2)} \propto \langle n_p \rangle^{-1} \)

The dependence of the expansion rate coefficient, \( C_2 \), on plasma density is shown in Figure 3.30 along with the displacement coefficient, \( C_d \equiv d/V_a \). Here, I lower the density by changing the total number of electrons, \( N_{tot} \), injected into the trap. The physical size of the plasma is kept relatively constant at \( \langle L_p \rangle \approx 20 \) cm and \( R_p/R_w \approx 0.4 \), by using lower confining potentials, \( V_c \), for the lower density plasmas, and by radially expanding the initially narrow columns. The total number and confinement voltages for the two density extremes are: \( N_{tot} = 2.4 \times 10^9 \) with \( V_c = -100 \) V for the highest density of \( \langle n_p \rangle = 1.6 \times 10^7 \) cm\(^{-3}\); and \( N_{tot} = 3.3 \times 10^8 \) with \( V_c = -35 \) V for the lowest density of \( \langle n_p \rangle = 0.2 \times \) cm\(^{-3}\). (Note: The data used to obtain the two smallest densities in Figure 3.30 is the only original data in the entire thesis for which \( V_c \neq -100 \) V.)

Both the expansion rate and the displacement decrease with increasing
Figure 3.30: Expansion rate coefficient $C_2$ versus the average plasma density $\langle n_p \rangle$ for $\langle R \rangle > 20$. The transport decreases with density approximately as $C_2 \propto \langle n_p \rangle^{-1}$. For the same plasmas, the fit coefficient for the center-of-mass displacement $C_d$ also decreases with density approximately as $d \propto \langle n_p \rangle^{-1}$. For the data in this experiment $N_L \propto \langle n_p \rangle$.

density approximately as $C_2, C_d \propto \langle n_p \rangle^{-1}$. The decrease of displacement with density is expected from Equation 3.15, since in these experiments $N_L \propto \langle n_p \rangle$. On the other hand, the observed decrease in the expansion rate with density is somewhat puzzling. Standard rotational pumping theory predicts a rate that is independent of plasma density, but this scaling assumes the perturbation does not change with density. It is unclear whether the perturbation can be considered fixed in these experiments, since the plasma distorts (by moving off-axis) a larger amount for lower density values. There is a need for more work on rotational pumping theory appropriate for an applied asymmetry, so that a more definitive
Figure 3.31: Expansion rate fit coefficient, $C_2$, versus the average plasma length $\langle L_p \rangle$ for $\langle R \rangle \gtrsim 20$. The measured expansion rate is roughly independent of plasma length, but the fit coefficient for the center-of-mass displacement $C_d$ varies as $C_d \propto \langle L_p \rangle^{-1}$.

3.6.6 Independence of Length: $\Delta \nu_{(r^2)} \propto \langle L_p \rangle^0$

In Figure 3.31, the expansion rate coefficient $C_2$ is shown to be roughly independent of the plasma length. These results disagree with standard rotational pumping theory, which predicts an $L_p^{-2}$ dependence for the expansion. The independence of $C_2$ is particularly puzzling in light of the measured center-of-mass displacement, which decreases with length approximately as $C_d \propto \langle L_p \rangle^{-1}$ as expected from Equation 3.15. In other words, even though the plasma is distorted (in this case, shifted off-axis) by a larger amount, the expansion rate does not increase
for shorter plasmas.

The discrepancy between the length dependencies for rotational pumping theory and the measured expansion rate may be due to a theory assumption which is not satisfied by the experiments. Rotational pumping theory is appropriate for a long, thin (pencil-like) plasma, where the aspect ratio is large $L_p/2R_p \gg 1$. However, for the smallest lengths in Figure 3.31 the aspect ratio is $L_p/2R_p \approx 1$, and the plasma is shaped more like a football than a pencil.

The circular point in Figure 3.31 is for a plasma confined entirely within the sectored ring. In this case, the asymmetry is nearly over the entire plasma ($k_z = 0$) and represents a departure from the standard experimental setup where the asymmetry is applied to one end of the plasma. Further results with this (and other non-standard experimental setups) are discussed in the following section.

### 3.7 Transport from Non-Standard Asymmetries

In this section, I present expansion rate measurements for:

- Asymmetries applied over nearly the entire plasma ($k_z = 0$): slightly and highly-rigid regimes.
- Asymmetries applied near the middle of the plasma: slightly-rigid.
- Two asymmetries applied at opposite ends of the plasma: slightly and highly-rigid.
- Time varying asymmetries with frequency $f_a < f_{diss} < f_E$: slightly-rigid.

The experiments presented in this section are different from my standard experimental setup consisting of a static asymmetry applied to one end of the plasma.
3.7.1 Asymmetry over the Entire Plasma \((k_z = 0)\)

I apply an asymmetry over nearly the entire axial length of the plasma by confining the electrons within the sectored ring only; voltages applied to the sectors of this ring can be considered to be nearly \(z\)-independent (\(i.e. k_z = 0\)). For example, in CamV I confine the plasma entirely within the S4 cylinder (see Figure 2.1) with L2 and H3 used together as the inject gate and H5 and G6 used as the dump gate. With confining voltages of \(V_c = -100\) V, the plasma length is \(\langle L_p \rangle \approx 3.5\) cm, and the perturbation length is calculated to be \(L_{pert} \approx 3.2\) cm. \((L_{pert}\) is smaller than \(\langle L_p \rangle\) because the azimuthal sections, which are 3.94 cm long, are not centered in the S4 frame.\)

For a \(k_z = 0\), \(m = 1\) asymmetry, the expansion rate is shown in Figure 3.32 to increase with asymmetry strength approximately as \(\Delta \nu_{(z)} \propto V^2\) for both slightly-rigid \((\langle R \rangle = 6.4)\) and highly-rigid \((\langle R \rangle = 220)\) plasmas. Calculations from the simple formula \(\Delta \nu_{(z)} = 7V_a \langle R \rangle^{-2}\) are shown to not agree with the measurement even for the slightly-rigid \((\langle R \rangle = 6.4)\) plasma.

It appears that the “\(V_a R^{-2}\)” transport mechanism is not effective when the asymmetry is applied over the entire length of the plasma. In other words, different mechanisms dominate the transport for a \(k_z = 0\) and a \(k_z \neq 0\) asymmetry, when applied to a slightly-rigid plasma. This is an indication that bounce-resonant particles contribute strongly to the “\(V_a R^{-2}\)” mechanism, since the resonance condition (Equation 3.46) cannot be satisfied for \(k_z = 0\) (see Section 3.10.1).

In the highly-rigid regime, however, there does not appear to be a difference in the asymmetry-induced transport for a \(k_z = 0\) and \(k_z \neq 0\) applied asymmetry. This similarity is shown in the plot of \(C_2\) versus \(\langle L_p \rangle\) (Figure 3.31) from the last section. In this figure the value of the expansion rate fit coefficient for \(k_z = 0\) (solid circular point) is roughly equal to the values for \(k_z \neq 0\) (solid square points).
Figure 3.32: Expansion rate versus voltage for a $k_z = 0$ perturbation. When the asymmetry is applied over nearly the entire plasma (schematic shown in top figure) the expansion rate is proportional to $V_a^2$ for both a slightly-rigid and highly-rigid plasma. The empirical scaling for slightly-rigid plasmas is shown by the solid lines.
3.7.2 Asymmetry Applied near the Middle

For a slightly-rigid plasma, the measured transport due to a $k_z \neq 0$ asymmetry does not appear to depend upon whether the asymmetry is applied near the middle of the plasma or to the end.

In Figure 3.33, the measured values of $\Delta \nu_{(r,z)}$ for $m = 1$, $m = 2$, and $m = 4$ asymmetries applied near the middle of the plasma are shown to be within a factor of 2 of the simple scaling found for asymmetries applied to the end of the plasma.

The radial dependence of the transport when the perturbation is applied near the middle of the plasma (not shown), is similar to the dependence for a perturbation at the end of plasma (shown in Figures 3.19, 3.20, and 3.21.) This similarity is true, even for an $m = 1$ perturbation, which can cause substantial transport at the center of the plasma. In other words, the plasma does not appear to shield an asymmetry applied near the middle of the plasma, anymore than it shields an asymmetry applied to the end.

For a highly-rigid plasma, the expansion rate is typically larger when the asymmetry is applied near the middle than at the end, but these results are preliminary, and are not presented here.

3.7.3 Asymmetries at Both Ends

By applying asymmetries at both ends of the plasma, I show that the expansion is not directly dependent on the center-of-mass displacement, $d$. This lack of correlation between $\Delta \nu_{(r,z)}$ and $d$ is found to occur in both the slightly-rigid and highly-rigid regimes. In the experiments, I vary the relative angle $\Delta \theta$ of the two perturbations. One simple result is that a second perturbation applied “out of phase” ($\Delta \theta = \pi$) with the first, reduces the displacement, but increases the expansion rate.
Figure 3.33: Expansion rate versus $V_a \langle R \rangle^{-2}$ for an asymmetry applied near the middle of a slightly-rigid plasma (schematic shown in top figure). The transport follows the same simple scaling $\Delta \nu = 7 V_a^2 R^{-2}$ used to describe the induced transport from an asymmetry applied to the end of the plasma.
Figure 3.34: Schematics of two asymmetries applied to opposite ends of the plasma. When the perturbations are “in phase” the relative angle is $\Delta \theta = 0$, and when they are “out of phase” the relative angle is $\Delta \theta = \pi$.

These experiments were performed on CamV by applying a nominal $m = 1$ asymmetry on the S4 sector and either a $m = 1 \pm$ or an $m = 1 \pm \pm$ on the S7 sector (see Appendix A for schematics of these perturbations). These are the only experiments in this thesis in which I apply an asymmetry to the S7 sector. I vary the relative angle, $\Delta \theta$, between the two asymmetries by using different sectors on S7.

I plot the expansion rate and the center-of-mass displacement, as a function of the relative angle between the two perturbations: for a slightly-rigid plasma in
Figure 3.35: Expansion rate and center-of-mass displacement versus the relative angle, $\Delta \theta$, between two asymmetries at opposite ends of the plasma in the slightly-rigid regime.

Figure 3.35 and for a highly-rigid plasma in Figure 3.36. The horizontal dashed lines indicate the values of $d$ and $\Delta \nu_{^{(2)}}$ for the case with only the asymmetry applied to S4.

In both figures, $d$ depends upon $\Delta \theta$ as expected, being a maximum at a small relative angle, and nearly zero when the perturbations are approximately $\Delta \theta = \pi$ radians out of phase. The expansion rate does not exhibit the same dependence on the relative angle as does the displacement.

In the slightly-rigid regime, shown in Figure 3.35, the expansion rate is shown to, in fact, be near a maximum where $d$ is at a minimum ($\Delta \theta \approx \pi$), and to be near a minimum where $d$ is at a maximum ($\Delta \theta \approx 0$); this is indicative of
enhanced transport due to a first order \((m = 1)\) resonance. In addition, \(\Delta \nu_{(r^2)}\) also peaks at \(\Delta \theta \approx \pi/3\) and is at a minimum near \(\Delta \theta \approx \pi/6\); this may be an indication of higher order \((m > 1)\) resonances (see Section 3.10.1).

In the highly-rigid regime, shown in Figure 3.36, the expansion rate near \(\Delta \approx \pi/4\) is nearly 3 times larger than the rate measured with just a single asymmetry applied on S4 only. In this case, relatively large values of \(\Delta \nu_{(r^2)}\) correspond to relatively large displacements. However, as \(\Delta \theta\) increases from approximately \(\pi/2\) to \(\pi\) the displacement decreases substantially below the “on S4 only” line, while the expansion rate remains about a factor of 2 larger than “on S4 only”. In other words, by adding a second perturbation, out of phase with the first, I decrease the

Figure 3.36: Expansion rate and center-of-mass displacement versus the relative angle, \(\Delta \theta\), between two asymmetries at opposite ends of the plasma in the highly-rigid regime.
Figure 3.37: Expansion rate versus frequency of an applied \( m = 1 \) asymmetry which varies in time as \( V_a(t) = 1 \text{ V} \sin(2\pi f_a t) \). The expansion rate is independent of \( f_a \) for frequencies below the \( m = 1 \) diocotron mode frequency, \( f_{d\text{doc}} \).

displacement, but increase the expansion rate.

3.7.4 Non-Static Asymmetry (\( f_a \neq 0 \))

In Figure 3.37, the measured expansion rate induced by a time varying \( m = 1 \) asymmetry,

\[ V_a(t) = 1 \text{ V} \sin(2\pi f_a t), \]

(3.25)

is plotted as a function of the asymmetry frequency \( f_a \). The measured expansion rate is roughly independent of the applied frequency for \( f_a < f_{d\text{doc}} < f_E \), where \( f_{d\text{doc}} \) is the frequency of the \( m = 1 \) diocotron mode. For applied frequencies near
\( f_a = f_{\text{dio}} \), the diocotron mode is excited and the measurements become excessively noisy.

For all experimental frequencies, the asymmetry is applied for \( t_{\text{pert}} = 25 \text{ ms} \), meaning that an increase in the frequency equates to an increase in the number of wave cycles applied (shown by the scale at the top of the graph). For the lowest frequency, \( f_a = 20 \text{ Hz} \), the asymmetry is applied for 1/2 cycle and can be thought of as a static perturbation which is ramped on and off in a sinusoidal form.

The data in Figure 3.37 shows that there is no substantial difference between a static asymmetry and an asymmetry at relatively small frequencies \( f_a < f_{\text{dio}} \). Frequency dependent transport at relatively high frequencies (\( f_a > f_E \)) has been seen to occur in other experiments, where the applied asymmetries drive axial standing waves in the plasma [30, 38].

### 3.8 Comparisons to Other Experiments

In this section, I compare the transport measurements from an applied electric asymmetry on CamV and EV to prior experiments at UCSD on transport due to inherent trap asymmetries on 5 different machines, to experiments on applied magnetic asymmetries on EV, and to experiments on damping of the \( m = 1 \) diocotron mode on EV. For slightly-rigid plasmas \( (1 < \mathcal{R} < 10) \), these prior experiments all exhibit transport rates which scale as \( \mathcal{R}^{-2} \), indicating that the \( V_e \mathcal{R}^{-2} \) mechanism is quite pervasive.

In this section, I also relate my measurements to experiments on asymmetry-induced transport conducted at the University of California, Berkeley by John Notte and Joel Fajans, and at Occidental College by Dennis Eggleston. These experiments mostly agree with my measurements, although some discrepancies remain.
Figure 3.38: Expansion rate versus rigidity for transport due to inherent trap asymmetries in EV \((V_o = 0)\). The transport rate is shown to follow the proportionality \(\nu_{(x)} \propto \langle R \rangle^{-2}\), with an overall magnitude determined by whether the S6 sectored-ring is included in the confinement region or not.

### 3.8.1 Transport due to Inherent Trap Asymmetries

Background transport due to the inherent asymmetries of the trap has been called “anomalous” transport, because the source and strength of the asymmetry is unknown. In this section, the transport is unambiguously identified with field-asymmetry transport exhibiting the same \(R^{-2}\) scaling found for an applied asymmetry. In addition, I present examples of “measurements” of the effective trap asymmetry strength, for EV, which are written in Table 3.1.
<table>
<thead>
<tr>
<th></th>
<th>EV w/ S6</th>
<th>EV w/o S6</th>
<th>Occidental (Ref. [25])</th>
<th>Berkeley (Ref. [50])</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>0.4 V</td>
<td>0.06 V</td>
<td>$\sim 1$ V</td>
<td>3 V</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.07 cm</td>
<td>0.01 cm</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 3.1:** Estimates for the effective trap asymmetry, $V_t$, and effective misalignment, $\Delta$, for the inherent asymmetries in EV with and without the sectored ring S6. Also listed are estimates of $V_t$ for electron traps at Occidental College and U. C. Berkeley.

**EXPANSION RATE ON EV**

Measurements of the expansion rate due to inherent trap asymmetries in EV are shown in Figure 3.38. The two different symbols represent data taken with and without the sectored ring (S6) included in the confinement. In both cases, the transport roughly follows the familiar scaling $\nu_{\langle r^2 \rangle} \propto \langle R \rangle^{-2}$. The transport is greater when the sectored ring is present, presumably since this ring is more asymmetric than all the other rings.

The effective strength of the inherent asymmetries in EV (both with and without S6) are calculated using $\nu_{\langle r^2 \rangle} = 7V_t \langle R \rangle^{-2}$ and the “eyeball” data fits shown in Figure 3.38. The calculated values for $V_t$ are displayed in Table 3.1 along with values for the effective misalignment (or trap error) distance $\Delta$, which is calculated using Equation 3.17 with $N_L = 5 \times 10^7$ cm$^{-1}$. Also listed in this table are estimates of $V_t$ for electron traps at Occidental College and U. C. Berkeley.

The trap asymmetries induce transport primarily at the edge of the plasma regardless of whether a sectored ring is included in the confinement region. Examples of the radial dependence of the change in density $\Delta n_c(r)/\Delta t$ are shown for the EV inherent asymmetries: with S6 in Figure 3.19(a) and without S6 in Figure 3.39. In both cases, the density near the center of the plasma remains relatively constant over the short duration of the experiment (i.e. $\Delta t \ll 1/\nu_{\langle r^2 \rangle}$). On a longer time
Figure 3.39: Change in density versus radial position for transport due to inherent trap asymmetries for the case without a sectored ring in the confinement region. The induced transport is primarily at the edge of the plasma.

scale, the central density decreases; I believe this transport is related to viscous transport, as discussed in Section 3.4.5.

The lack of transport at the center of the plasma, for inherent asymmetries in EV, is most likely due to the asymmetry being partially shielded from the center of the column, as was seen for an $m = 1$ asymmetry at $V_a < 1$ Volt (see Section 3.4.5). Recall, however, that the quantity $\nu_{(c^2)}$ is dominated by the transport at the edge of the column, where the asymmetry does not appear to be shielded.

**TRANSPORT ON 5 DIFFERENT TRAPS**

Transport due to inherent trap asymmetries has been studied for over 20 years and on 5 different Penning-Malmberg traps here at UCSD. Many of the
previous studies measured the “mobility time” $\tau_m$, defined as the time for the central density $n_0(t) \equiv n(r = 0, t)$ to decrease to $1/2$ its initial value, i.e.,

$$n_0(\tau_m) \equiv n_0(0)/2$$

(3.26)

**EV and $V'$**

In 1983 Driscoll and Malmberg reported a surprising length dependence to the transport as measured on the (now decommissioned) $V'$ machine [15]. The mobility time was found to scale as $\tau_m \propto (B/L)^2$ over more than 5 decades in $\tau_m$. Later measurements performed on EV found the same scaling for $\tau_m$, but at about a factor of 20 slower [14]. The reduced transport in EV was attributed to the greater care taken in reducing the construction asymmetries. (Note that most of this EV data was taken without the S6 sectored ring.)

In Figure 3.40, I plot the inverse of the $\tau_m$ measurements for EV and $V'$ as a mobility rate, $\tau_m^{-1}$, versus the ratio $B/L$. The scale on the top of the graph is a crude estimate of the initial rigidity at the center of the plasma $R_0 \equiv R(r = 0)$ using Equation 2.19 and the approximations $n_o \equiv n_e(r = 0) \approx 10^7 \text{ cm}^{-3}$ and $T \approx 1 \text{ eV}$. This rigidity calculation is an underestimate of the time-averaged rigidity over the duration $\tau_m$, because as the plasma expands the rigidity of the plasma increases from its initial value by about a factor of 3. The increase in rigidity is due to both a decrease in density (by $1/2$) and an increase in temperature (by about a factor of 2).

In Figure 3.40 the data sets for the two machines are shown to follow the approximate scalings:

$$\tau_m^{-1} = 3.1 (B/L)^{-2} \approx 0.07 R_0^{-2} \quad \text{for EV}$$

(3.27)

$$\tau_m^{-1} = 63 (B/L)^{-2} \approx 1.3 R_0^{-2} \quad \text{for $V'$}.$$
Figure 3.40: Mobility rate versus $B/L$ for EV and $V'$. The transport rates due to inherent trap asymmetries in the two machines scale as $\tau_m^{-1} \propto (B/L)^{-2} \propto R_0^{-2}$, where $R_0$ is an estimate of the initial rigidity at the center of the plasma.

The EV scaling of $\tau_m^{-1}$ with rigidity is somewhat lower than my measured scaling for $\nu_{(b,y)}$ (without the S6 ring) for two reasons: (1) The quantity $\nu_{(b,y)}$ is dominated by the transport at the edge, which changes faster than the central density for inherent asymmetries in EV. (2) The average rigidity over the evolution of the plasma $\langle R \rangle$ is used in the scaling of $\nu_{(b,y)}$, but the initial rigidity at the center of the plasma $R_0$ (which is smaller than $\langle R \rangle$) is used in scaling $\tau_m$. In other words, the quantity $R_0$ is not an accurate description of the plasma parameters over the relatively long durations of the $\tau_m$ experiments.
Figure 3.41: Mobility rate versus rigidity for inherent asymmetries on CamV. The rate $\tau_m^{-1}$ is proportional to $\mathcal{R}_0^{-2}$ for $1 < \mathcal{R}_0 < 10$ and independent of $\mathcal{R}_0$ for $\mathcal{R}_0 > 10$. The EV (solid) and $V'$ (dashed) scalings are drawn for comparison. The dotted horizontal line is an estimated prediction for transport in the highly-rigid regime.

CamV

Measurements of the mobility rate $\tau_m^{-1}$ taken on CamV by Cass and Fine are shown in Figure 3.41. The rates scale as $\mathcal{R}_0^{-2}$ in the slightly-rigid regime, $(1 < \mathcal{R}_0 < 10)$ with an overall magnitude that agrees with the $V'$ measurements (dashed line). I crudely estimate the trap asymmetry to be $V_t \sim 1$ V for both CamV and $V'$, since the CamV and $V'$ inherent transport measurements are about a factor of 20 larger than the EV measurements (in which $V_t \approx 0.06$ V without S6).

In the highly-rigid regime ($\mathcal{R}_0 \gtrsim 20$) the transport rate due to inherent
asymmetries in CamV is shown to be independent of the rigidity. The transition between a rate that decreases as $\mathcal{R}^{-2}$ and a rate that is independent of $\mathcal{R}$ occurs in the same rigidity range ($\mathcal{R} = 10 - 20$) as for the transition of $\Delta \nu_{(v,z)}$ due to applied asymmetries (Section 3.5).

The measurements of $\tau_m^{-1}$ do not exhibit a dramatic decrease in magnitude in conjunction with the change in parameter scaling, as is seen for an applied asymmetry. This can be partially explained by the fact that the density and temperature are relatively low ($n_0 \approx 0.2 \times 10^{-7}$ cm$^{-3}$, $T \approx 0.5$ eV) for the CamV $\tau_m^{-1}$ measurements at high rigidity. A crude estimate of transport in the highly-rigid regime is shown as the dotted line in Figure 3.41; this estimate was calculated using the empirical formula for $\Delta \nu_{(v,z)}$ given in Equation 3.23 with $V_a = V_i = 1$ V and the above values of density and temperature.

**CV and IV (electrons or ions)**

Using the “CV” (Cryogenic) machine, Cluggish measured the dependence of $\tau_m$ on density and temperature. The measurements show that the transport rate increases strongly with density and generally decreases with temperature. The basic parameter values for the CV plasmas were $T = 0.1 - 10$ eV, $n = 10^9 - 10^{10}$ cm$^{-3}$, $L \approx 5 - 10$ cm, and $B = 4 - 60$ kGauss.

In Figure 3.42, I plot the CV mobility rate $\tau_m^{-1}$ versus (a) $B/L$ and (b) $\mathcal{R}_0$. As was shown in reference [5], the rigidity is a much better indicator of the transport rate than just $B/L$ alone. The solid and dashed lines are respectively the EV and $V'$ scalings shown in Figure 3.40 and written in Equations 3.27 and 3.28; and the extent of the lines shows the extent of the data.

In Figure 3.42, I have also plotted measurements by Hollmann and Anderson conducted with the “IV” (Ion) device. The measurements are of $\nu_{n_0}$ for
Figure 3.42: Transport rates versus (a) $B/L$ (b) $R_0$ for CV and IV (electrons or magnesium ions). The ion data is scaled by the square root of the mass ratio. The solid and dashed lines are the scalings for EV and $V'$ respectively. The transport rates on the different lines are in rough agreement when plotted as a function of rigidity. Both CV and IV (ion) data become independent of $R_0$ for $R_0 \lesssim 1$. 
electrons \((B = 40 \text{ kGauss}, L_p = 35 \text{ cm}, T = 0.2 \text{ eV}, n_0 = 10 - 140 \times 10^7 \text{ cm}^{-3})\) and 
\(\nu_{(p)}\) for magnesium ions \(^{24}\text{Mg}^+ \ (B = 40 \text{ kGauss}, L_p = 12 \text{ cm}, T = 0.004 - 4 \text{ eV}, n_0 = 0.3 - 14 \times 10^7 \text{ cm}^{-3})\).

The quantities \(\tau_m^{-1}, \nu_{n_0}, \text{ and } \nu_{(p)}\) are all transport rates, which should exhibit the same parameter scalings and roughly the same magnitude. To compare the measurements for magnesium ions to measurements for electrons, I multiply the ion rates by the factor \(\sqrt{M_{Mg}/m_e} \approx 218\), where \(M_{Mg}\) is the mass of a magnesium ion. The mass-ratio factor is motivated by the fact that the equations of motion and Maxwell’s equations are identical for electrons and singly-ionized ions if time is scaled by the square root of the mass ratio [12]. The mass also appears in the rigidity through the bounce frequency, meaning that ions are less rigid by a factor of \(\sqrt{M_{Mg}/m_e}\) than electrons at the same \(B, L, T, \) and \(n\).

For slightly-rigid \((1 < R_0 < 10)\) plasmas, the transport rates as measured on CV and IV are within a factor of 10 of either the EV or \(V'\) scalings when plotted as a function of rigidity. Taken together, transport rates due to inherent trap asymmetries have been measured on 5 different machines here at UCSD, and the vast majority of the data has a rigidity in the range \(1 < R < 10\). This slightly-rigid data is well described by the scaling

\[
\text{Transport Rate} \times \sqrt{M/m_e} \sim (0.1 - 1.0) R^{-2} \propto \frac{L^2 n^2}{B^2 T M}. \quad (3.29)
\]

For floppy \((R_0 < 1)\) plasmas the transport rate is found to be independent of the plasma rigidity for electrons in CV and for Ions in IV as shown in Figure 3.42(b). Cluggish found that by just lowering the temperature the expansion rate \(\tau_m^{-1}\) changed from \(\tau_m^{-1} \propto R_0^{-2} \propto T^{-1}\) for \(R_0 > 1\) to \(\tau_m \propto R^0 \propto T^0\) for \(R_0 \ll 1\). This data set is shown as the larger diamonds in Figure 3.42.
The CV and IV data indicate that the transport process in the slightly-rigid regime, which was found to “turn off” for large rigidity \( R \gtrsim 20 \), also turns off at low rigidity \( R \lesssim 1 \). But in the EV and \( V' \) data shown in Figure 3.40 no such turn off for \( R < 1 \) was seen. Instead, the transport rate for EV and \( V' \) continues to increase in the floppy regime for reported values of the rigidity as low as \( R_0 \sim 0.1 \). While the rigidity values given for the EV and \( V' \) data are most likely under-estimations of the average rigidity in the plasma, a key difference may be the higher collisionality in the colder, denser CV and IV plasmas.

3.8.2 Magnetic Tilt Asymmetry

A “magnetic tilt” is a misalignment of the magnetic field compared to the axis of the trap cylinder. This magnetic asymmetry induces transport in a slightly-rigid plasma that scales as \( \langle R \rangle^{-2} \). By comparing the transport rates, I find that a 1 mrad magnetic tilt in EV is equivalent to a 0.2 Volt asymmetry applied to the trap wall.

The magnetic field can be tilted by using the \( B_x \) and \( B_y \) saddle coils, which are typically used for aligning the field. A magnetic tilt has an azimuthal dependence of \( m = 1 \) with an axial dependence of \( k_z = 1 \). Fine studied the effects of a magnetic tilt using the EV apparatus by measuring the rate of change of the angular momentum

\[
\nu_c \equiv -\frac{1}{L_\theta} \frac{dL_\theta}{dt}.
\]

The tilt angle, shown schematically in Figure 3.43 and defined as

\[
\Theta_B \equiv \Delta B / B,
\]

was varied from \(-0.3 \text{ mrad} \leq \Theta_B \leq 2.5 \text{ mrad} \) for a range in magnetic field of \( B = 50 - 300 \text{ Gauss} \) and a range in confinement length of \( L = 19.8 - 35.6 \text{ cm} \). To
Magnetic Tilt

Electric Tilt

Figure 3.43: Schematic of a “magnetic tilt” and an “electric tilt”. For a magnetic tilt, the plasma tilts relative to the trap axis. For an electric tilt, the equilibrium position in the trap (dashed line) is effectively tilted relative to the plasma.

get an idea of how tilt transport scales with $B$, Fine measured the slope $d
\nu_{\perp}/d\Theta_B$ near $\Theta_B = 0$. In Figure 6.9 of Fine’s thesis [31], data at $L = 35.6$ cm shows that

$$
\frac{d\nu_{\perp}}{d\Theta_B} \approx 300 \left[ \frac{B}{100 \text{ G}} \right]^{-2}
$$

over the range $B = 50$ to $300$ G. A length scaling was not reported by Fine, but he does say, “...the effect of the tilt is much stronger in longer plasmas” [31].
I rewrite Equation 3.31 in terms of \( \nu_{(r^2)} \) and \( \langle \mathcal{R} \rangle \) using

\[
\nu_c = \frac{\langle r^2 \rangle}{R_w^2 - \langle r^2 \rangle} \nu_{(r^2)}
\]

along with the relationship between \( \mathcal{R} \) and \( B \) (Equation 2.19) and the approximate experimental values: \( \langle r^2 \rangle \approx 2.5 \text{ cm}^2 \), \( \langle n_p \rangle \approx 5 \times 10^6 \text{ cm}^{-3} \), \( T \approx 1.5 \text{ eV} \), \( \langle L_p \rangle \approx 30 \text{ cm} \) and \( R_w = 3.81 \text{ cm} \). Fine’s data thus suggests that a magnetic tilt of angle \( \Theta_B \) causes transport at a rate

\[
\Delta \nu_{(r^2)} \approx 1.4 \left[ \frac{\Theta_B}{\text{mrad}} \right] \langle \mathcal{R} \rangle^{-2}
\]  

(3.32)

for slightly-rigid plasmas in the range \( 0.5 < \langle \mathcal{R} \rangle < 3 \). I have taken some data on tilt transport in the range \( \langle \mathcal{R} \rangle = 1 \) to 4, and find that my 4 data points all agree with the above scaling within a factor of 2.

By comparing Equation 3.32 to the equation for transport due to an applied voltage \( (\Delta \nu_{(r^2)} = 7 \nu_{a} \langle \mathcal{R} \rangle^{-2}) \), I find the following equivalence between a magnetic tilt, \( \Theta_B \), and an applied asymmetry voltage, \( \nu_{a} \), in EV:

\[
\frac{\Theta_B}{1 \text{ mrad}} = \frac{\nu_{a}}{0.2 \text{ Volt}}
\]

If the plasma is in the highly-rigid regime, presumably one could translate Equation 3.23 for a magnetic asymmetry to obtain

\[
\Delta \nu_{(r^2)} = 2 \times 10^{-5} \text{ sec}^{-1} \left[ \frac{\Theta_B}{1 \text{ mrad}} \right]^2 \left[ \frac{\nu_{a}}{1 \text{ eV}} \right]^\frac{1}{2} \left[ \frac{10^7 \text{ cm}^{-3}}{\langle n_p \rangle} \right] \text{ speculative.}
\]

where again \( \beta = 1/2 \) for \( T < 1 \text{ eV} \) and \( \beta = 1 \) for \( T > 1 \text{ eV} \). However, this empirical formula has not yet been tested, and a corresponding equivalence between magnetic and electric tilts in the highly-rigid regime has not yet been investigated.

To provide a further comparison between a magnetic tilt and an electric asymmetry, I define an “electric tilt angle” shown schematically in Figure 3.43 and defined as

\[
\Theta_E \equiv \frac{\Delta}{L_p},
\]  

(3.33)
where $\Delta$ is the shift in the equilibrium due to an $m = 1$ asymmetry.

CamV images demonstrate that the plasma response to an $m = 1$ asymmetry is a shift relative the the axis of the cylinder and not a tilt (Figure 3.9). Instead of a tilt of the plasma, $\Theta_E$ represents a tilt in the equilibrium position as shown in Figure 3.43 (also see Figure 3.12). In both schematics of an electric and magnetic tilt, the electrons are simply taken to follow the magnetic field lines as they bounce in $z$.

Using Equation 3.17 for $\Delta$, the electric tilt angle is related to the strength of a nominal $m = 1$ applied asymmetry as

$$\Theta_E \approx 40 \text{ mrad} \left( \frac{R_w}{L_p} \right) \left[ \frac{V_a}{1 \text{ Volt}} \right] \left[ \frac{5 \times 10^7 \text{ cm}^{-1}}{N_L} \right].$$

(3.34)

Combining this equation with the scaling $\nu_{y^2} = 7 V_a \langle R \rangle^{-2}$, and using the EV value $R_w = 3.81 \text{ cm}$, I obtain

$$\Delta \nu_{y^2} \approx 0.46 \left[ \frac{\theta_E}{\text{ mrad}} \right] \langle R \rangle^{-2} \left[ \frac{L_p}{10 \text{ cm}} \right] \left[ \frac{5 \times 10^7 \text{ cm}^{-1}}{N_L} \right].$$

(3.35)

Finally, by using the experimental values for the Fine data of $N_L = 10^8 \text{ cm}^{-1}$ and $\langle L_p \rangle = 30 \text{ cm}$, I arrive at a formula appropriate for comparison to Equation 3.32 as

$$\Delta \nu_{y^2} \approx 3 \left[ \frac{\theta_E}{\text{ mrad}} \right] \langle R \rangle^{-2}.$$

Thus, according to these simple calculations: In the slightly-rigid regime, a magnetic tilt and an electric tilt cause transport of nearly the same magnitude (within a factor of 2), and have the same approximate scaling with rigidity, $\Delta \nu_{y^2} \propto \langle R \rangle^{-2}$.

### 3.8.3 Damping of the $m = 1$ Diocotron Mode: Slightly-rigid

The $m = 1$ diocotron mode is essentially the orbit of a column which has been “bopped” off-axis by an amount $D$. This differs from the case of an applied
asymmetry, where the column is held in place off-axis, and is static in the lab frame (see Section 3.3).

The damping rate of the diocotron mode is defined as

$$\gamma \equiv -\frac{1}{D} \frac{\partial D}{\partial t}. \quad (3.36)$$

As discussed in Section 3.2 and written in Equation 3.3, the mean-square radius calculated from the center of the trap can be written as $\langle r^2 \rangle = \langle \rho^2 \rangle + D^2$. Taking the time derivative of this relationship, I obtain the following equation, which is proportional to the change in the angular momentum

$$\frac{d\langle r^2 \rangle}{dt} = \frac{d\langle \rho^2 \rangle}{dt} - 2\gamma D^2 \propto \frac{dL_\theta}{dt}. \quad (3.37)$$

If angular momentum is conserved, the column expands as the mode damps (*i.e.* $D$ decreases while $\langle \rho^2 \rangle$ increases.)

Cluggish performed excellent measurements of the damping of the diocotron mode on the CV apparatus at relatively high magnetic fields [6, 5]. He found that the angular momentum was conserved in the process and that the theory of rotational pumping [8] (described further in Section 3.10.2) does a remarkably good job at describing his data over a large range in plasma parameters, including over more than 3 decades in temperature.

There are two terms to the theoretical damping from rotational pumping: an adiabatic term $\gamma_{adb}$ and a resonant particle term $\gamma_{res}$. The resonant particle term is predicted to enhance the damping rate near $R \sim 1$ and is proportional to $R^{-5}$. Cluggish found that his data agreed extremely well with the adiabatic term, $\gamma_{adb}$ alone, even near $R \sim 1$. He concluded that the relatively high collisionality of the plasma prevented resonances from occurring, where the condition for resonant particle theory to apply can be approximated as

$$\nu_{\parallel} \ll \bar{f}_e. \quad (3.38)$$
Rotational pumping is discussed in more detail in Section 3.10.2 in comparison to the asymmetry transport measurements in the highly-rigid regime.

Mitchell measured the damping of the diocotron mode and subsequent expansion of the plasma column on EV for displacement values of $0 < d < 0.4$. This data can be found in Figure 3.8 of Reference [47]; however, a word of caution: the quantity listed as $\gamma$ in the table of this figure is actually $\gamma R_w^2$.

The data was fit to the curves

$$\nu_{(\rho^2)} = A + 2\gamma \frac{D^2}{\langle \rho^2 \rangle} = A + \gamma d^2.$$  \hspace{1cm} (3.39)

Angular momentum was not conserved in these experiments; however, the loss in angular momentum was found to be independent of $D$. In the above equation, $A$ represents this loss, and is just equal to the transport from inherent trap asymmetries (i.e. $A = \nu_{bg} = \nu_{(\rho^2)}$). Therefore, the plasma expansion given by $\nu_{(\rho^2)}$ increases has two terms: one due to inherent asymmetries and one due to the damping of the diocotron mode.

Mitchell found that measured values of $\gamma$ are from 6 to 38 times larger than the adiabatic version of rotational pumping $\gamma_{adh}$, and do not follow the predicted parameter scalings. It appears that another process dominates over adiabatic rotational pumping.

I have re-analyzed the Mitchell data in terms of the rigidity and find that both the background expansion $A$ and the additional expansion caused by the damping of the diocotron mode $\tilde{\gamma}$ are roughly proportional to $R_0^{-2}$ as shown in Figure 3.44.

Cluggish calculated a theoretical estimate of rotational pumping for this EV data using the sum of the adiabatic and resonant particle terms multiplied by
Figure 3.44: Expansion due to damping of the diocotron mode in EV. For the data taken by Mitchell, both the background expansion rate and the expansion due to the damping of an $m = 1$ diocotron mode are roughly proportional to $R_0^{-2}$.

an end slant correction

$$\gamma_{\text{theory}} = \left( \frac{1.5 \sqrt{R_w/R_p}}{j_{01}} \right)^2 (\gamma_{adb} + \gamma_{res}), \quad (3.40)$$

where $j_{01} \approx 2.40$. This estimate is shown by the solid curve in Figure 3.44. It exhibits an approximate $R_0^{-5}$ scaling from the resonant particle term. For this EV data, the inequality in Equation 3.38 does indeed hold, and resonant particle theory should apply. However, it is clear that this estimate of the theory does not agree with the EV measurements.

I believe that a mechanism other than rotational pumping is responsible for the damping of diocotron mode and subsequent expansion in EV. The fact that the scaled damping rate is proportional to $R_0^{-2}$ makes me believe that the mechanism
is intimately related to the mechanism responsible for transport from an externally applied (or inherent trap) asymmetry.

3.8.4 Measurements at Berkeley

In addition to studying the equilibrium shape, Notte and Fajans measured \( \tau_m \) due to an applied asymmetry at Berkeley. For the most part, their experimental results can be explained by the empirical scalings found for my measurements. For the Berkeley experiments, the asymmetry was applied to one sector near the end of the plasma, for both negative and positive values in the range \( V_a = 2 - 40 \) V.

The plasma parameters were varied around the values \( R_p \approx 1.0 \) cm, \( B = 500 \) Gauss, \( n_0 = 10^7 \) cm\(^{-3} \), \( T \approx 1.6 \) eV, and \( L_p \approx 10 \) cm. I calculate a rigidity for these parameter as \( \mathcal{R} = 9.2 \), which is near the transition from the slightly-rigid to highly-rigid regimes. For most of the data, the rigidity is increased above \( \mathcal{R} = 9.2 \) as one parameter is varied.

They report the following scalings [50],

\[
\tau_m^{-1} \propto n_0^{3.3} B^{-0.65} T^0 \left( V_{a}^{2} + V_{t}^{2} \right),
\]

(3.41)

where \( V_t \approx 3 \) V is the reported estimate for the inherent trap asymmetry. They state that the length dependence was not clear from the data.

The voltage dependence is reportedly the most robust result, and agrees with my voltage scalings for \( \langle \mathcal{R} \rangle > 10 \).

The magnetic field dependence \( (\tau_m^{-1} \propto B^{-0.65}) \) was found by varying the field over the range \( B = 500 \) to 2000 G, which I calculate as a range in rigidity of \( \mathcal{R} = 9.2 \) to 37. The weak dependence on the magnetic field could be the effect of the majority of the data being in the highly-rigid \( (\nu_{\gamma, 2} \propto B^{0}) \) regime with a few data points in the slightly-rigid \( (\nu_{\gamma, 2} \propto B^{-2}) \) regime.
The density dependence \( \tau_m^{-1} \propto n^{3.3} \) was found by varying the density as well as the asymmetry strength so that the product of the two remained fixed (\( n_0 V_a = \text{constant} \)). The density was varied over the range \( n_0 = 0.4 - 3 \times 10^7 \text{ cm}^{-3} \), which I calculate as a rigidity range of \( R = 3 \) to \( 30 \). A rough \( n^3 \) scaling is not inconsistent with my results in the slightly-rigid \( (\nu_{(a)} \propto \langle n_p \rangle^2) \) regime, but does disagree with the density dependence in the highly-rigid \( (\nu_{(a)} \propto \langle n_p \rangle^{-1}) \) regime. The fact that the voltage was not kept fixed makes the comparisons questionable at best.

The most puzzling result is the \( \tau_m^{-1} \propto T^0 \) scaling. This dependence was found for the range \( T = 1.6 - 6.4 \text{ eV} \), which I calculate as a rigidity range of \( R = 9.2 \) to \( 18 \). In both the slightly-rigid and highly-rigid regimes, I find that the transport decreases with temperature approximately as \( \nu_{(a)} \propto T^{-1} \) for \( T > 1 \text{ eV} \). In addition, for both IV and CV data (with \( R_0 > 1 \)), \( \tau_m^{-1} \) was found to decrease with temperature [5, 37].

Some differences between my results and the Notte-Fajans results may be due to differences in experimental techniques. They measured the change in the central density for long-time evolutions, where a large portion of the plasma is lost to the walls, whereas I measure the global expansion over relatively short times. In addition, Notte and Fajans describe the central density evolution with two time scales. At first the plasma expands rapidly without loss of charge over a time \( \tau_1 \). Then “...the expansion slows down as the plasma approaches the chamber walls ...” [50]. They ignore the initial density decay in their definition of \( \tau_m \). However, they do measure \( \tau_1 \) as a function of the applied voltage and report the interesting proportionality

\[
\tau_1^{-1} \propto V_a^{-1}. \tag{3.42}
\]

They do not mention the plasma parameters used to obtain this linear
scaling, and I am left to assume that the rigidity is \( R = 9.2 \). If this is the case, then I can explain their results as follows: The plasma is slightly-rigid to begin with and expands as \( \tau_1^{-1} \propto V_a^{-1} \). As the plasma expands, the density decreases and the temperature increases causing an increase in the rigidity (i.e. \( R > 10 \)). The (now) highly-rigid plasma then expands further as \( \tau_m^{-1} \propto V_a^2 \).

Recently, the Berkeley group has begun to study transport induced by an applied quadrupole (\( m = 2 \)) magnetic asymmetry. Gilson and Fajans have preliminary results that indicate the transport rates scale as \( B^{-2} \), and is enhanced due to bounce-resonant particles [35].

### 3.8.5 Measurements at Occidental

Dennis Eggleston is currently studying asymmetry-induced transport of “test particles” in a rotational flow [25]. He reports loss rates due to inherent trap asymmetries (which he estimates to be \( V_i \sim 1 \) Volt [25]) that are surprisingly similar to mobility rates on EV.

The experimental setup at Occidental College closely resembles EV, with one major difference: the plasma has been replaced by a wire along the axis of the trap. This wire is biased negatively (simulating the space-charge of the absent plasma) and a small number of electrons are injected off center. These electrons form an annulus of charge, which \( \mathbf{E} \times \mathbf{B} \) drift around the wire. Confinement principles are the same as for a column of electrons.

Instead of measuring \( \tau_m \), Eggleston measures \( \tau_{1/2} \), the time for the total number of electrons \( N_{\text{tot}}(t) \) to decrease by 1/2,

\[
N_{\text{tot}}(\tau_{1/2}) \equiv \frac{1}{2} N_{\text{tot}}(0). \tag{3.43}
\]

Eggleston’s measurements track the EV \( \tau_m^{-1} \) measurement remarkably well as a
function of the ratio \((B/L)\) \[25\], \textit{i.e.}

\[
\tau_{1/2}^{-1} \approx 3.1(B/L)^{-2}.
\] (3.44)

In other words, the transport rates for test-particles exhibit similar parameter scalings to an electron plasma. This similarity is especially remarkable in light of the fact that the density of the test particles is so low that \(\lambda_D > R_w\), and therefore, the test-particles should not exhibit collective phenomena.

There are puzzling differences in the dependence on rotation frequency and density. For his test-particles, Eggleston finds in separate experiments that:

- the transport rate is independent of \(f_E\)
- the transport rate is independent of \(n\).

The rotation frequency \(f_E\) of the test particles is set by the bias on the wire. When this bias is varied a factor of 5 (from -30 to -150 Volt), Eggleston finds that \(\tau_{1/2}\) only changes by 30%. This result conflicts with the theoretical idea that the \(\nu_{\rho} \propto (n/B)^2\) scaling for my measurements is due to the rotation frequency \(f_E\). However, Eggleston also varied the density by a factor of 10 and found no more than a 30% change in \(\tau_{1/2}\); this is in disagreement with my approximate empirical scaling \(\nu_{\rho} \propto n^2\) scaling, and is also in sharp disagreement with the strong density dependence of \(\tau_m\) as seen by Cluggish in CV \[5\].

Eggleston has recently begun studying the effects of an applied (non-static) asymmetry. The 5 confinement rings of the trap are each divided into 8 azimuthal sections. He applies asymmetries of a nearly pure axial mode number as well as azimuthal mode number. He finds a strong dependence on the applied frequency, \(f_a\), with a local flux that peaks at different frequencies for different radial locations. In general the flux peaks at a relatively high frequency, \(f_a \sim 1\) MHz. He reports
that his preliminary results are in qualitative agreement with resonant particle transport theory [26, 29].

Eggleston measures the dependence of the transport on the voltage strength by measuring the local change in density of the test particles $dn(r)/dt$. He adjusts the frequency to maximize the flux at one particular radial position in the trap, $R$, and then varies the voltage of this oscillating signal. Eggleston finds that the change in the density at $R$ varies as

$$\frac{dn(R)}{dt} \propto V_a^{2.1} \text{ for } 0.03 \leq V_a \lesssim 0.2$$

$$\propto V_a^{1.3} \text{ for } 0.2 \lesssim V_a \lesssim 3. \quad (3.45)$$

This change in scaling from roughly $V^2$ to $V^1$ shows some resemblance to the voltage dependence of my measurements of the rate of change of the central density, $\nu_n$, in the slightly-rigid regime as discussed in Section 3.4.5. However, a key difference may prove to be the relatively high frequencies used by Eggleston. I anticipate that further measurements at both UCSD and Occidental will bridge the remaining empirical gaps between asymmetry-induced transport on test-particles and on non-neutral plasmas.

## 3.9 Summary

The major results of this chapter are summarized by Figure 3.45. In this figure, the expansion rate at $V_a = 1$ Volt, as given by $C_1$ (Equation 3.18) or $C_2$ (Equation 3.24), is plotted versus the average plasma rigidity. The measurements are divided up into the regimes: slightly-rigid, $1 \lesssim \langle R \rangle \lesssim 10$, and highly-rigid, $\langle R \rangle \gtrsim 20$.

In the slightly-rigid regime, the expansion rate decreases with rigidity as $\langle R \rangle^{-2}$ and is linearly dependent on the applied voltage, $V_a$. The unknown mech-
Figure 3.45: Expansion rate versus rigidity showing two transport regimes. The parameter dependence of the expansion rate is distinctively different in the two regimes, which are differentiated from each other by the plasma rigidity.

Anism responsible for this so-called $V_a \mathcal{R}^{-2}$ transport appears to “turn-off” for a rigidity in the range $\langle \mathcal{R} \rangle = 10 - 20$. Here, the transport for a 1 Volt asymmetry drops precipitously as a different process dominates for $\langle \mathcal{R} \rangle > 20$. In the highly-rigid regime, the transport is independent of the plasma rigidity and scales as $V_a^2$.

3.9.1 Slightly-Rigid Regime

In the slightly-rigid regime the major experimental results are:

- The expansion rate depends linearly on the applied voltage: $\Delta \nu_{r,2} \propto V_a$.
- The expansion rate decreases with rigidity: $\Delta \nu_{r,2} \propto \langle \mathcal{R} \rangle^{-2}$.
• The global expansion rate does not depend upon the azimuthal mode number of the asymmetry.

• The local flux does depend upon the azimuthal mode number as \( \Gamma \propto r^m \).

Other results are that the applied asymmetries do not appear to be shielded, and are well approximated by the vacuum potentials. This is evident by the local flux measurements, which follow the proportionality \( \Gamma_x(r) \propto r^m \) even near the center of the plasma. This is surprising, because the electrostatic vacuum potential due to the asymmetry is less than the thermal energy within the plasma (i.e. \(|e \Phi_a(r, \theta)| \ll T \) for \( r \leq R_p \)). The vacuum potentials are particularly small near the center of the plasma \( (r \ll R_w) \), yet substantial transport is still observed to occur here. This lack of shielding occurs for asymmetries applied either to the end (Figure 3.1) or near the middle of the plasma column (Figure 3.33).

The transport decreases when the perturbation is applied over the entire length of the plasma, indicating that there must be an axial dependence to the asymmetry (i.e. \( k_z \neq 0 \)) for the so-called “\( V_o \mathcal{R}^{-2n} \)” mechanism to be effective.

The transport is independent of the plasma distortion, which is induced by the applied asymmetry. For example, the expansion rate of the plasma is shown to not directly depend upon the center of mass displacement for two \( m = 1 \) perturbations applied to opposite ends of the plasma.

The expansion rate is also shown to be independent of the frequency of the applied perturbation. This result indicates that the plasma is not resonantly driven by a mode.

The “\( V_o \mathcal{R}^{-2n} \)” transport mechanism caused by an applied asymmetry, seems to be the same mechanism responsible for the background transport due to slight construction errors. An \( \mathcal{R}^{-2} \) scaling (previously written as \( (B/L)^{-2} \) for constant
is shown to describe the measured transport rates due to inherent trap asymmetries on 5 different machines.

The effective trap errors are the same order of magnitude as the applied asymmetries. On EV, which is generally considered to have the weakest inherent asymmetries, I estimate the effective trap asymmetry to be $V_t = 0.4$ Volt with the sectored ring and $V_t = 0.06$ Volt without. The effective trap asymmetries estimated from transport measurements at other institutions are $V_t \sim 1$ Volt at Occidental and $V_t \approx 3$ Volt at Berkeley.

The $R^{-2}$ scaling is also shown to hold for transport due to an applied magnetic asymmetry and due to damping of the $m = 1$ diocotron mode. In addition, a magnetic tilt is shown to give nearly the same transport as an effective electric tilt from an $m = 1$ applied asymmetry.

For the case of test-particles, as measured at Occidental, the background transport rate is found to scale as $(B/L)^{-2}$, but strangely does not depend upon the density or rotation frequency of the particles. Preliminary measurements of asymmetry-induced transport, show a roughly $V_a^2$ dependence at low voltage, which changes to a roughly $V_a^1$ dependence for $V_a > 0.2$ Volt. These measurements along with preliminary measurements at Berkeley on applied magnetic asymmetries show some agreement with theories of transport due to bounce-resonant particles.

The “turn-off” of the $V_a R^{-2}$ mechanism, as shown by the precipitous drop and change in parameter dependence in Figure 3.45, is indicative of the bounce motion becoming a good adiabatic invariant in the range $R = 10 - 20$. Breaking the bounce adiabatic invariant is necessary for a change in thermal energy (Joule heating), which must occur when the electrostatic energy decreases as the plasma expands [5]. Collisions between electrons can break this invariant, but so can “collisions” with an asymmetric field. For the field to break the invariant it must
vary in time (as seen by the electron) at a rate on the order of or greater than the bounce frequency. In the frame of the rotating electron, the field varies at the \( \mathbf{E} \times \mathbf{B} \) rotation frequency; therefore, if the bounce motion is sufficiently greater than the rotation (\textit{i.e.} sufficiently large rigidity) the asymmetry does not break the bounce adiabatic invariant (\textit{i.e.} it is a “good” adiabatic invariant), and irreversible transport must rely on electron-electron collisions.

The transport mechanism in the slightly-rigid regime remains a mystery. The rigidity scaling, taken together with evidence from other experiments, indicate that resonant particles are important; however, no current version of resonant particle theory predicts the observed \( V_a^1 \) scaling. These theories have progressed by considering the asymmetry to be small and using a perturbative approach, but it appears that even the inherent trap asymmetries are too large for such an approach to be appropriate. In this case, simulations may prove useful in attempts to provide the long-awaited theoretical explanation of this pervasive transport mechanism.

3.9.2 Highly-Rigid

The main experimental results in the highly-rigid regime are

- The expansion rate increases with the applied voltage as \( \Delta \nu_x \propto V_a^2 \).
- The expansion rate is not directly dependent on the rigidity.
- The expansion rate is roughly independent of magnetic field and length, and inversely proportional to the density and temperature.

Other results are that unlike the slightly-rigid regime, transport in the highly-rigid regime does not decrease when the perturbation is applied over the entire plasma.
With two $m = 1$ asymmetries applied to opposite ends of the plasma, the expansion rate was shown to not directly depend upon the center of mass displacement.

Previous experiments on asymmetry-induced transport at Berkeley primarily in the highly-rigid regime, found a roughly $V_a^2$ scaling and a weak magnetic field dependence, in agreement with the scalings I report. The density and temperature dependence disagree somewhat, and questions remain about the effects of different experimental techniques.

“Rotational pumping” of axial energy is the likely mechanism, responsible for transport in the highly-rigid regime. However, calculations of rotational pumping appropriate for “football” shaped plasmas are needed for a more definitive comparison.

### 3.10 Directions for Theory

#### 3.10.1 Bounce-Resonant Particles

Many aspects of the observed transport in the slightly-rigid regime agree with theories of enhanced transport due to bounce-resonant particles. However, these theories do not predict the observed linear dependence on asymmetry strength. In addition, the theory work has primarily advanced in a regime, which may be inappropriate for the experiments.

The strong dependence on the plasma rigidity and the “turn-off” for $k_z = 0$ are both indications that bounce resonances are important for transport in the slightly-rigid regime. A particle which is bounce-resonant with an applied perturbation of the form $\Phi_0 \propto e^{i\pi k_z z/L_p} e^{im\theta}$ satisfies the condition

$$ m f_E = k_z f_b. $$

(3.46)
A resonant particle encounters the same portion of the asymmetry field upon each bounce, and can therefore take many radial steps in the same direction.

Resonant particle theory calculations [29] are divided into two different regimes (I do not consider the stochastic regime here):

- **Plateau Regime**: \( \nu_{\text{eff}} > \omega_{tr} \)
- **Banana Regime**: \( \nu_{\text{eff}} < \omega_{tr} \),

where \( \nu_{\text{eff}} \) is the effective collision frequency and \( \omega_{tr} \) is the trapping frequency of particles in the field of the asymmetry.

The effective collision frequency is estimated as [29]

\[
\nu_{\text{eff}} \approx \nu_{ee} \left( \frac{\bar{v}}{\Delta v} \right)^2,
\]

where \( \Delta v \) is the resonance width, given by \( \Delta v = \nu_{\text{eff}} (L_p/\pi k_z) \) in the plateau-regime, and by \( \Delta v \approx \omega_{tr} (L_p/\pi k_z) \) in the banana regime.

I estimate \( \omega_{tr} \) as

\[
\omega_{tr} = \frac{\pi k_z}{L_p} \left( \frac{e \Phi_m}{m_e} \right)^{1/2}
\]

(3.48)

using the vacuum potentials given in Equation 3.10 along with \( k_z = 1 \). I feel that using these unshielded potentials to calculate \( \omega_{tr} \) is appropriate for two reasons:

1. The global expansion rate, \( \nu_{(z,r)} \), and thus the change in angular momentum, is dominated by the transport at the edge of the plasma (see Equation 3.4), where shielding is weakest.
2. Flux measurements are proportional to \( r^m \) indicating that the asymmetry “penetrates” far into the plasma for voltages as low as \( V_a = 1 \) Volt. Evidence for shielding (Section 3.4.5) is only seen near the very center of the plasma at small applied asymmetries.

I find that for the experiments presented in this chapter, the plasma is in the “banana-regime” (i.e. \( \nu_{\text{eff}} < \omega_{tr} \)). This condition holds even at the lowest applied
voltages, \((V_a = 0.1 \text{ V})\) as well as for the estimated trap errors \((V_t \sim 0.1 - 1 \text{ V})\), except near the very center of the plasma.

Calculations in the “banana-regime” are difficult, since the effective collision frequency is so low that particles can take effectively large radial steps before being knocked out of resonance. The particle orbits are highly nonlinear, and a perturbative approach is not appropriate. However, a simple heuristic calculation for the resonant-particle flux in the banana-regime can be made, as given in Reference [29]. Simplifying this calculation by assuming a gradient-free plasma, I find the approximate parameter dependence for the flux as

\[
\Gamma_m \propto \frac{n}{T} \left( \frac{m f_E}{f_b} \right)^2 \frac{\Phi_{a,m}^{1/2}}{r} \left( \frac{r}{R_w} \right)^{m/2},
\]

where the flux is for a single \(m\) number perturbation. This equation is remarkably close to the empirical model given in Equation 3.21, with the main difference being the \(\Phi_{a,m}^{1/2}\) dependence instead of the observed \(\Phi_{a,m}^3\) dependence. I believe that a more advanced theory of resonant-particle transport in the banana regime, would be useful in attempts at describing the \(V_a R^{-2}\) transport mechanism.

Resonant particle theory in the plateau-regime is more advanced than in the banana-regime, but the expansion rate measurements presented in this chapter are in sharp disagreement with predictions from this theory. In the plateau-regime, frequent collisions mean that the effective step size is small and a perturbative approach is appropriate. Calculations of the flux in this regime are independent of the collision frequency and are proportional to \(\Phi_{a,m}^2\).

### 3.10.2 Rotational Pumping

Rotational pumping was briefly discussed in Section 3.8.3 in relation to the damping of the \(m = 1\) diocotron mode. In that section, I presented data on slightly-rigid plasmas in EV which strongly disagree with rotational pumping
theory. In this section, I compare predictions from the theory to the results on expansion due to an applied asymmetry in the highly-rigid regime.

In rotational pumping [8, 6], the pumping comes about due to a difference in the plasma length, $\delta L$, from one side of the column compared to the other. An electron’s parallel energy is pumped as it rotates about the center of the plasma column. This change in parallel energy is irreversibly scattered into perpendicular energy, and there is a net increase in the thermal energy. This increase in thermal energy comes at the expense of electrostatic energy, which decreases by way of plasma expansion.

In the adiabatic limit (appropriate for highly-rigid plasmas), the expansion rate due to rotation pumping has the following approximate parameter dependence:

$$\nu_{rot.} \propto n^0 B^0 T^{-1/2} \left(\frac{\delta L}{L}\right)^2.$$

(3.50)

The most noteworthy dependence is the actual lack of dependence on the magnetic field strength.

For an $m = 1$ diocotron mode, the length change $\delta L$ comes about due to off-axis plasma being confined by the roughly parabolic confining potentials. It is believed that an asymmetry applied to the end of the plasma causes a length change by reflecting electrons at a different axial position on one side of the column, where a negative potential is applied, compared to the other, where a positive potential is applied. In this case, $\delta L$ is taken to be proportional to $V_a$, although this has not been verified in any capacity. The plasma is also displaced off axis for an $m = 1$ applied asymmetry, but the expansion rate was shown to not depend directly on this displacement.

The measured expansion rate due to an applied asymmetry in the highly-rigid regime is roughly independent of magnetic field, which is a strong indication
that rotational pumping is the dominant mechanism. The voltage and temperature
dependencies are also consistent with rotational pumping; however, the density and
length dependencies are not. Numerical comparisons were not made, due to a lack
of knowledge of $\delta L$ in this case. Solutions to Poisson’s equation in three-dimensions
may be useful in determining an estimate for $\delta L$, but this has not yet been done.

In addition to the expansion due to an applied asymmetry, I also measured
the damping of the diocotron mode in a highly-rigid plasma. The damping rates
for a short “football-shaped” plasma are not in agreement with simple rotational
pumping theory predictions. The measured damping rate $\gamma$ is about a factor of 20
less than $\gamma_{adb}$.

The disagreement of the measured damping rates with rotational pumping
for short CamV plasmas, as well as the disagreement with the length and den-
sity dependence for expansion due to an applied asymmetry, could be due to the
shape of the plasma. The experimental plasmas are somewhat spheroidal for the
measurements I conducted, with $L_p \gtrsim 2R_p$, whereas the simple rotational pump-
ing theory assumes a pencil-like plasma having $L_p \gg 2R_p$, as was the case for
the previous rotational pumping experiments in CV [6, 5]. It is unclear what
an appropriate evaluation of rotational pumping theory would predict for a more
spheroidal plasma. There is a need for calculations in this geometry so that a
better comparison can be made.
Chapter 4

Viscous Transport

4.1 Overview

In this chapter, I present measurements of viscous transport in pure-electron plasmas. Viscous transport causes the plasma to evolve toward the global thermal equilibrium state of rigid rotation and essentially uniform density. Figure 4.1 shows the evolution of the radial density and rotation profile. The plasma is initially created with substantial radial density gradients and strong rotational shear. The electrons in the plasma rearrange themselves to eliminate the shears; as a consequence, some electrons move radially inward while others move outward. This radial transport is well described by a local model of viscosity, which depends upon the coefficient of viscosity and on the shear in the plasma.

I experimentally measure the viscosity coefficient and make detailed comparisons to predictions from theories of transport due to like-particle collisions. I find that there is no substantial difference between viscous transport in a monotonic or hollow profile, as was previously believed. Both the magnitude and parameter scaling of the measured viscosity disagree dramatically with Classical theory predictions. Theories of Long-Range $\mathbf{E} \times \mathbf{B}$ drift collisions provide much more accurate predictions. However, no current version of these theories sufficiently describes
Figure 4.1: Example of density profile evolution due to viscous transport. The density profile smooths out, as the plasma evolves to a near rigid rotation. The angular momentum of the plasma is conserved (i.e. \( \nu_\theta x_\theta = 0 \)) in the process indicating an internal like-particle process.

the data. I present an empirical formula for the viscosity; this formula includes finite length effects and is basically a simplified hybrid of a 3D and a 2D version of Long-Range theory. The empirical formula is shown to provide a good description of the entire range of measured viscosity values in Figure 4.16.

In Section 4.2, I discuss the standard model of viscous transport in a cylindrical fluid; this model forms the basis for comparisons between experiments and theories. The model relates the viscous stress in the plasma to the radial transport of particles. Here, the stress, \( P_{r\theta} \), is defined to be proportional to the product of the coefficient of viscosity, \( \eta \), and the shear in the total fluid rotation as \( P_{r\theta} = -\eta r \partial \omega_{\text{tot}} / \partial r \).
Theories which are used to calculate viscosity coefficients are discussed in Section 4.3. In a pure-electron plasma the Debye length is larger (typically much larger) than the cyclotron radius, \( \lambda_D \gg r_c \). The Classical theory of transport only includes velocity-scattering collisions between electrons separated by less than or equal to the cyclotron radius, \( \delta \sim r_c \). More recent theories by Dubin and O’Neil predict much larger viscosity coefficients by considering long-range Coulomb collisions over distances up to a Debye length \( \delta \leq \lambda_D \). In these theories, electrons take radial steps by \( \mathbf{E} \times \mathbf{B} \) drifting due to their mutual Coulomb interaction. I describe the 3D version of these so called Long-Range theories, along with predicted 2D enhancements to the viscosity. One 2D enhancement applies only to plasmas with a hollow rotation profile (i.e. rotation peaks off center), while a more recent theory applies to both hollow and monotonically decreasing profiles.

In Section 4.4 I discuss and display previous measurements of the transport to thermal equilibrium. These experiments measured the global rate at which a short, initially hollow plasma evolved toward equilibrium. This equilibrium rate, \( \nu_{eq} \), is much larger and scales differently with magnetic field than Classical theory predictions. The experiments were not sufficiently detailed to measure a local coefficient of viscosity, or for that matter, even verify whether viscous forces were responsible for the observed transport. However, the measured scaling of \( \nu_{eq} \) with magnetic field implies a viscosity coefficient proportional to \( B^1 \), which disagrees with the \( B^0 \) magnetic field dependence of the 3D Long-Range theory. These experiments motivated the 2D Long-Range theory, which predicts an enhancement for hollow profiles.

In Section 4.5, I describe the measurements of local radial transport shown in the remainder of the chapter. I present the method used to calculate the local coefficient of viscosity, \( \eta_e(r) \), from just the experimental density profiles, \( n_e(r) \), and
the measured plasma temperature, $T$.

In Section 4.6, the measured local flux of particles, $\Gamma_x(r)$, and experimental stress, $P_x(r)$, are shown to be well described by the model of viscous transport, in which the transport is driven by shears in the total fluid rotation. This shear-driven model is shown to describe the measurements for monotonic as well as hollow profiles.

In Section 4.7, the measured kinematic viscosity, $\kappa_x \equiv \eta_x / m_e n_c$, is shown to roughly increase with magnetic field as $\kappa_x \propto B^1$ and to decrease with length as $\kappa_x \propto L^{-1}$. Thus, the viscosity increases with the plasma rigidity roughly as $\kappa_x \propto R \propto B/L$, although there is substantial scatter in the data. The magnetic field dependence is displayed in Figure 4.12, where the Classical theory is shown to be as much as $10^4$ times less than the measurements and scales differently with magnetic field. The measured viscosity agrees much better with Long-Range theories than with the Classical theory. The 3D version agrees with the data at relatively low values of the rigidity (i.e. low magnetic field and long plasma length). However, this theory is independent of $R$, and is more than a factor of 10 smaller than the data at the highest rigidity. This enhancement above the 3D prediction, is indicative of a 2D finite length effect, since it roughly increases with rigidity.

In Section 4.8, I compare the measured viscosity to predictions from the version of 2D Long-Range theory that is appropriate for both monotonic and hollow plasmas. The 2D theory is shown to be about a factor of 10 larger than the measurements. However, simplifying approximations used in the theory calculations are not well satisfied by the experiments.

In Section 4.9, viscosity data is shown in comparison to a simple empirical formula, which depends upon a finite-length effect given by the average number of collisions experienced by interacting electrons. This empirical form for the viscosity
Figure 4.2: Fluid element in a shear flow.

is a hybrid of the 3D and 2D *Long-Range* theories, and is shown to describe the full range of data in Figure 4.16.

In the final section of this chapter (Section 4.11), I present an example of a plasma in which both asymmetry and viscous transport can be seen. The total radial flux of the plasma is well described by the sum of the two empirical models given in this thesis.

### 4.2 Model of Local Viscosity

In this section, I describe the standard model of viscous transport in a cylindrically-symmetric fluid. This model provides a basis for theoretical and experimental definitions of the coefficient of viscosity for a pure-electron plasma. A cartoon example of a fluid element in a shear flow is shown in Figure 4.2. Here, a fluid element experiences a radial drift, $v_r$, due to a viscous force, $F_\theta$, where the force is proportional to the gradient in the stress, $P_{r\theta}$.

In describing the viscous model, I borrow the approach taken by Driscoll
in Reference [17] by beginning with the force-balance equation for a fluid of charge \(-e\) and density \(n\),
\[
\nabla \cdot \mathbf{P} = -e n \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right),
\]
(4.1)
where \(\mathbf{P}\) is the pressure-stress tensor. The diagonal elements of this tensor are equal to the pressure (\(i.e.\ P_r = n T\)), and the off-diagonal elements are symmetric and represent the stresses presumed to be solely due to viscous interactions. Here, I am concerned with the radial transport due to shears in the azimuthal velocity, \(v_\theta(r)\); in this case, viscosity causes a \(r, \theta\) component of the stress tensor,
\[
P_{r\theta} = -\eta r \frac{\partial \omega_{\text{tot}}}{\partial r},
\]
(4.2)
where \(\eta\) is the coefficient of viscosity and \(\omega_{\text{tot}} = v_\theta / r\) is the total fluid rotation frequency in the azimuthal direction. In the remainder of this chapter I will use the generic term “stress” to refer to the theoretical quantity in Equation 4.2, and the corresponding experimental quantity in Equation 4.27.

An expression for \(\omega_{\text{tot}}\), can be found by taking the \(r\)-component of Equation 4.1, which leads to
\[
\omega_{\text{tot}} \equiv \frac{v_\theta}{r} = \frac{c}{B r} \left[ \frac{\partial \phi}{\partial r} - \frac{1}{e n} \frac{\partial (n T)}{\partial r} \right].
\]
(4.3)
The first term on the right side is identified as the \(\mathbf{E} \times \mathbf{B}\) drift frequency, \(\omega_E\), and the second term is the diamagnetic (or pressure) drift, \(\omega_D\).

By taking the \(\theta\)-component of Equation 4.1 and using the relationships \(P_{r\theta} = P_{br}\) and \(P_{r\phi} = 0\), I arrive at an expression for the radial flux of particles in terms of the stress as
\[
\Gamma \equiv n v_r = \frac{c}{e B r^2} \frac{\partial}{\partial r} \left( r^2 P_{r\theta} \right)
= \frac{c}{e B r^2} \frac{\partial}{\partial r} \left( r^2 \eta r \frac{\partial \omega_{\text{tot}}}{\partial r} \right).
\]
(4.4)
Previous experiments on like-particle collisional transport measured the rate of global relaxation toward thermal equilibrium, $\nu_{eq}$, as a function of magnetic field. This equilibrium rate is proportional to the flux, and from the above equation the magnetic field dependence of the flux (and thus $\nu_{eq}$) can be seen to be $1/B^2$ times the magnetic field scaling of the viscosity coefficient, i.e.

$$\text{magnetic field dependence : } \nu_{eq} \propto \frac{\eta}{B^2}. \quad (4.5)$$

This equation is used to determine the implied magnetic field dependence of the viscosity from measurements of $\nu_{eq}$.

To theoretically calculate the coefficient of viscosity, one must go beyond the fluid model and consider interactions at the particle level. The viscosity coefficient, in effect, describes angular momentum exchange between interacting particles, and necessarily has the form

$$\eta = m_e n \nu_{eff} \delta^2, \quad (4.6)$$

where $\nu_{eff}$ is the effective collision frequency of momentum exchange, and $\delta$ is the distance over which the electrons interact. In comparing theoretical predictions to experimental measurements I often plot the kinematic viscosity $\kappa$, which is related to $\eta$ as

$$\kappa = \frac{\eta}{m_e n} = \nu_{eff} \delta^2. \quad (4.7)$$

In a local model, the transport equations depend upon the local plasma parameters. For a theta-symmetric column, the quantities in Equations 4.2 to 4.7 are more precisely written as two-dimensional functions of $r$ and $z$, e.g. $\tilde{P}_{\theta}(r,z)$ and $\tilde{\eta}(r,z)$. (Here I use the convention defined in Section 2.4, where "~" is used to denote a quantity that varies with $z$.) However, the experimental measurements are $z$-integrated, and thus only vary with the local radial position $r$. To provide
an appropriate comparison to the experiments, I consider the $z$-integral of the theoretical two-dimensional stress as

$$P_{r\theta}(r) = \frac{1}{L_c} \int_0^{L_c} dz \, \tilde{P}_{r\theta}(r, z)$$

$$= - \frac{1}{L_c} \int_0^{L_c} dz \, m_e \, \tilde{n}(r, z) \, \tilde{v}_{eff}(r, z) \, \tilde{\delta}^2(r, z) \, \frac{r \partial \omega_{tot}(r, z)}{\partial r}$$

$$\approx -m_e \, n_c(r) \, \nu_{eff}(r) \, \delta^2(r) \, r \partial \omega_{tot}(r)/\partial r,$$ (4.8)

where $\nu_{eff}(r)$ and $\delta(r)$ are calculated from the one dimensional plasma quantities, $n_p(r), L_\theta(r), \text{etc.}$. The last line is a good approximation, assuming local quantities do not vary substantially in $z$.

### 4.3 Theories of Viscous Transport

In this section, I briefly describe theories of electron-electron interactions and the values of the kinematic viscosity which they predict. While I find that no one theory describes all of my data, it is clear that predictions given by theories of Long-Range $E \times B$ drift interactions agree much better with the data than predictions from classical velocity-scattering collisions.

The theories predict fluxes which depend upon the derivative of the shear in the rotation frequency. A theoretical value of the coefficient of viscosity is defined by comparing these theoretical fluxes to the viscous flux given in Equation 4.4.

In this section, I find it convenient to express some quantities in terms of the simplified collision frequency

$$\nu_c \equiv n \, \bar{v} \, b^2.$$ (4.9)

For instance, the electron-electron collision frequency given by Equation 2.20 can be re-written as

$$\nu_{ee} = \frac{16 \sqrt{\pi}}{15} \nu_c \ln (r_c/b).$$ (4.10)
4.3.1 Classical Boltzmann Theory

Classical theory describes transport due to velocity-scattering collisions between electrons separated by a cyclotron radius or less. In Figure 4.3, a simple cartoon of the interaction shows that the guiding centers (+) of the cyclotron orbits take equal and opposite steps as the velocity vectors of the electrons scatter. The interaction occurs over a distance $\delta \leq r_c$, at rate of $\nu_{eff} \sim \nu_{ee}$.

A theory of radial transport appropriate for pure-electron plasmas [53], published by O’Neil and Driscoll in 1979, was constructed by expanding the Boltzmann equation in inverse powers of the magnetic field. From this work, the classical kinematic viscosity, $\kappa_{cl}$ can be found to be

$$\kappa_{cl} = \frac{3}{8} \nu_{ee} r_c^2. \quad (4.11)$$

Ignoring the logarithmic term in $\nu_{ee}$, this prediction has the following dependencies on the basic plasma parameters $B$, $L_p$, $T$, and $n$:

$$\kappa_{cl} \propto n \ T^{-1/2} \ L_p^0 \ B^{-2}. \quad (4.12)$$
Experiments have found that the transport toward thermal equilibrium, which is presumably driven by like particle interactions, strongly disagrees with this *Classical* theory. In the following sections, experimental values for both transport rate ($\nu_{eq}$) and kinematic viscosity ($\kappa_a$) are shown to be orders of magnitude larger than *Classical* theory predictions.

### 4.3.2 Long-Range $\mathbf{E} \times \mathbf{B}$ Drift Theory

Long-range like-particle interactions, as analyzed by Dubin and O’Neil, give substantially larger viscosity coefficients than *Classical* theory gives. In these *Long-Range* theories, a pair of interacting electrons move radially by $\mathbf{E} \times \mathbf{B}$ drifting in the presence of their mutual coulomb repulsion. In Figure 4.4, a cartoon example is shown to illustrate the mechanism.

These $\mathbf{E} \times \mathbf{B}$ drift collisions can take place over distances up to a Debye length, *i.e.* $\delta \leq \lambda_D$. The effective collision rate is essentially the simple collision frequency, *i.e.* $\nu_{eff} \sim \nu_e$. From the basic form of the viscosity given by Equation 4.6, it can be seen that in a non-neutral plasma (with $\lambda_D \gg r_e$) the kinematic viscosity due to long-range collisions will be much larger than for velocity scattering collisions.

The longer a pair of electrons interact, the greater the radial step they take. Thus “resonant” interactions are predicted to dominate the transport in these theories. In the 3D version of the theory, finite-length effects are ignored, and electrons which are resonant in their axial motion dominate the transport.

Disagreements between this 3D theory and experiments provided the impetus for a predicted 2D enhancement to the transport. This theory is 2-dimensional, since the bounce motion is averaged out and the electrons interact as rods of charge. In this case, transport is dominated by rods which are resonant in their azimuthal
rotation. The resonant interactions was first predicted to occur only for hollow plasmas, but the latest version of the theory allows for resonant interactions in both monotonic and hollow plasmas.

3D Long-Range Theory

In the 3D version of Long-Range $E \times B$ drift collisions, the kinematic viscosity has been recently re-calculated as [19]

$$\kappa_{3D} = \frac{\sqrt{\pi}}{3} \nu_c \ln \left( \frac{\bar{v}}{v_{\text{min}}} \right) \lambda_D^2.$$  \hspace{1cm} (4.13)

The recalculation is a factor of 2 larger than in the original 1985 paper on Long-Range collisions by O’Neil [52]. (Note: in the 1985 paper O’Neil gives a coefficient
$K$ which is related to the kinematic viscosity by $\kappa = (n/r^2) K$.

Ignoring the logarithmic term in Equation 4.13, the kinematic viscosity for
the 3D theory depends on plasma parameters as

$$\kappa_{3D} \propto n^0 T^{-1/2} L_p^0 B^0. \quad (4.14)$$

Thus, for a fixed temperature, the kinematic viscosity predicted from 3D theory
is approximately constant, regardless of changes in the magnetic field, length, or
density.

This transport is predicted to be dominated by electrons which are nearly
resonant in their axial motion. Equation 4.13 is calculated from an integral over
the relative parallel velocity $\Delta v_{||}$ of two interacting electrons, and small relative
velocities lead to large $E \times B$ steps due to the long duration of the interaction. To
avoid a logarithmic divergence at zero relative velocity, physical effects must be
taken into account which limit the time of the interaction. In effect, the velocity
integral is cut off at some small (but finite) value $v_{\text{min}}$, leading to the logarithmic
term in Equation 4.13.

Two different effects may contribute to this velocity cut-off: shear separa-
tion in the azimuthal direction and velocity diffusion in the axial direction. In
both cases, a minimum velocity $v_{\text{min}}$ is found by calculating the time $\tau$ for the two
interacting electrons to separate by a distance of $\lambda_D$. Theoretically, $v_{\text{min}}$ is set by
the effect which limits the interaction first, \textit{i.e.} the smaller value of $\tau$, which gives
the larger value of $v_{\text{min}}$.

For shear separation, the time and velocity are estimated to be [52, 19]:

$$\tau_{\text{shear}} = \frac{\lambda_D}{|r \Delta \omega_E|}$$

$$v_{\text{min}} \approx \lambda_D \tau \frac{\partial \omega_E}{\partial r} \quad (4.15)$$
where in the last line the approximation $\Delta \omega_E \approx \lambda_D \partial \omega_E / \partial r$ is used.

For velocity diffusion, the time and velocity are estimated to be [52]:

\[
\tau_{diff} = \frac{\lambda_D}{(\nu_{ee} \bar{v}^2 \tau_{diff})^{1/2}}
\]

\[
v_{\text{min}} = \left( \nu_{ee} \bar{v}^2 \lambda_D \right)^{1/3}.
\]

(4.16)

An enhancement to the kinematic viscosity by as much as a factor of 3 is predicted to occur when the electron interactions are exclusively limited by axial velocity diffusion [18]. This effect, known as “velocity caging”, occurs because electrons diffuse in velocity space until they eventually interact again. If no other effects (such as shear separation) decorrelate the electrons, each pair of colliding electrons will interact on average 3 times, leading to an enhancement of the viscosity by a factor of 3.

For the data presented in this chapter, velocity diffusion is calculated to occur slightly faster than shear separation, i.e. $\tau_{diff} \lesssim \tau_{\text{shear}}$. There is substantial theoretical question as to whether an enhancement due to caging should occur when the inequality $\tau_{diff} \ll \tau_{\text{shear}}$ is not strictly satisfied [19]. In calculating theoretical predictions for the 3D Long-Range theory, I do not include caging effects; rather I simply use Equation 4.13 with $v_{\text{min}}$ set by the maximum of either Equation 4.15 or 4.16.

The theories of electron-electron interactions due to long-range interactions also predict coefficients for heat transport and test-particle diffusion. These latter two coefficients have been measured by Hollmann, Anderegg, and Driscoll in pure ion plasmas, consisting primarily of singly-ionized magnesium, $^{24}\text{Mg}^+$. The Long-Range heat transport predictions agree with the measured thermal conductivity (within a factor of 2) over a wide range in plasma density, temperature, and magnetic field. The Long-Range theory of test-particle diffusion , which is closely
related to viscous transport, has also been found to agree well with the measurements. The original calculation was a factor of 3 less than the measurements over a wide range in plasma parameters. This prompted the re-analysis by Dubin which led to the discovery of the factor of 3 enhancement due to velocity caging. Note that (unlike in my viscosity measurements) in these experiments $\tau_{\text{diff}} \ll \tau_{\text{shear}}$, so that the velocity caging effect is unquestionably applicable.

2D Long-Range Enhancement

For large axial bounce frequencies, two electrons are believed to interact multiple times, producing a sequence of correlated $E \times B$ drift steps. These multiple interactions occur due to the finite length of the plasma, and are not considered by the 3D Long-Range theory. Dubin and O’Neil have devised two separate 2D theories which predict an enhancement to the total viscosity of the plasma. In these theories the electrons are bounce-averaged into “rods” of charge, which undergo 2D $E \times B$ drift transport.

In both the 2D theories, the transport is dominated by rods which move “resonantly” in the azimuthal direction. Rods are generally taken to move at the $E \times B$ drift velocity, in which case the resonance condition is $\omega_E(r_1) = \omega_E(r_2)$ for rods at $r_1$ and $r_2$.

All 2D $E \times B$ drift theories, necessarily give transport rates that scale with magnetic field as $\nu \propto B^{-1}$, which equates to a viscosity coefficient that scales as $\eta \propto B^1$. The earliest theories considered a neutral plasma, where the rotation frequency is taken to be zero ($\omega_E(r) = 0$), and the resonance condition $\omega_E(r_1) = \omega_E(r_2)$ is trivially satisfied everywhere in the plasma. However, in non-neutral plasma experiments $\omega(r)$ is not zero, nor is it typically uniform in radius. The two Dubin-O’Neil 2D theories consider two different ways by which the resonant
motion condition can be satisfied.

**Hollow Only**

The first 2D analysis by Dubin and O’Neil [20] concluded that significant transport would occur only for non-monotonic (i.e. hollow) $\mathbf{E} \times \mathbf{B}$ rotation profiles, such as was present in the original experiments [16]. It was believed that for interacting rods to be resonant, and thus cause significant transport, they must be at different radii and have the same $\mathbf{E} \times \mathbf{B}$ rotation frequency.

**Monotonic or Hollow–2D Long-Range Theory**

The second analysis by Dubin and O’Neil (which I refer to exclusively as the 2D Long-Range Theory) was motivated by my experimental result that the transport for monotonic rotation profiles is not substantially different than for hollow profiles. In this 2D theory, interacting rods can rotate resonantly even if $\omega_E(r)$ is not equal for the two rods. The analysis includes the extra velocity-dependent drift $\Delta \omega(r, v_\parallel)$ experienced by electrons when they are reflected by the confining potentials at the end of the column. (Such drifts have previously been considered in relation to the limits on 2D fluid studies [57] and the finite length frequency shift of the $m = 1$ diocotron mode [32]).

The bounced-averaged rotation frequency of the rods is taken to be

$$\omega_{r,0}(r, v_\parallel) = \omega_E(r) + \Delta \omega(r, v_\parallel),$$  \hspace{1cm} (4.17)

and the resonance condition becomes $\omega_{r,0}(r_1, v_{\parallel 1}) = \omega_{r,0}(r_2, v_{\parallel 2})$.

In Reference [22], $\Delta \omega(r, v_\parallel)$ is written as

$$\Delta \omega(r, v_\parallel) = -\frac{v_\parallel^2}{r \Omega_c L_p(r)} \frac{\partial L_p(r)}{\partial r}. \hspace{1cm} (4.18)$$

The more energetic electrons experience a larger drift for two reasons: they encounter the end fields more often due to their more rapid bounce motion, and
penetrate further into the confinement potentials.

The interaction distance between resonant rods, \( \delta_{2D} = r_2 - r_1 \), is estimated to be [22]

\[
\delta_{2D} = \frac{\bar{v}^2}{\Omega_c r L_p} \left| \frac{\partial L_p / \partial r}{\partial \omega_E / \partial r} \right|,
\]

this estimate is obtained by taking a Taylor expansion of Equation 4.17 around \( r_1 \), assuming \( \delta_{2D} \ll r_1 \), and using the approximation \( v_{||1}^2 - v_{||2}^2 \approx \bar{v}^2 \).

The predicted 2D \textit{Long-Range} kinematic viscosity is [22],

\[
\kappa_{2D} = 16\pi^2 \left( \frac{\int_0^\infty \! dt}{r \, \partial \omega_E / \partial r} \right) \nu_c \delta_{2D}^2 g(2\delta_{2D}/r).
\]

The factor \( \frac{\int_0^\infty \! dt}{r \, \partial \omega_E / \partial r} \) is identified as the average number of collisions experienced by electrons as they bounce between the ends, and is defined as \( N_{col} \) in Section 4.9. It can be thought of as an enhancement to the effective collision frequency. The function \( g(2\delta_{2D}/r) \) is an integral, which is calculated numerically. For the data in this chapter, I calculate the argument of this function to be \( 0.1 \lesssim (2\delta_{2D}/r) \lesssim 1 \), which means that \( g \) is in the range \( 1 > g > 0.1 \). (A table of values for \( g \) is given in Reference [22].)

To obtain the above formula for \( \kappa_{2D} \), Dubin and O’Neil assume the local interaction distance \( \delta_{2D} \) is much smaller than the Debye length and the radial position in the plasma, \( i.e. \)

\[
\delta_{2D} \ll \lambda_D \quad \text{and} \quad \delta_{2D} \ll r.
\]

From the definition of \( \delta_{2D} \), these inequalities are seen to be more easily satisfied at larger radial position, longer plasma length, and greater \( \mathbf{E} \times \mathbf{B} \) shear. For most of the experimental data, these inequalities are not strictly satisfied.

Ignoring the anomalous \( g \)-factor in Equation 4.20, the dependence of the 2D kinematic viscosity on the basic plasma parameters is

\[
\kappa_{2D} \propto n^1 T^1 L_p^{-3} B^1.
\]
4.4 Previous Measurements

Transport to thermal equilibrium was observed in 1988 by Driscoll et al. with the EV apparatus, using short hollow plasmas. The rate, $\nu_{eq}$, at which the plasma approached the equilibrium state was measured to be orders of magnitude faster than Classical theory predictions with an $\nu_{eq} \propto B^{-1}$ magnetic field scaling.

An example of the evolution towards equilibrium is shown in Figure 4.5. The $z$-integrated confinement density, $n_c(r)$, and the $z$-averaged total rotation frequency, $\omega_{tot}(r)$ are displayed at three different times. Also shown is the diamagnetic drift $\omega_D(r)$ at $t = 10$ sec. Initially, the density is peaked near the edge of the column, and the total rotation profile has substantial shear, with a peak in the total rotation rate near the plasma edge that is about 40% larger then the rotation at the center. As particles are transported radially, the density profile evolves such that the total rotation profile (not the $E \times B$ rotation profile) becomes essentially uniform. The temperature (not shown) was radially uniform throughout the evolution, but increased from about 0.8 eV at $t = 0.1$ sec to 1.1 eV at $t = 10$ s.

The relatively short columns used to obtain this data are observed to be experimentally stable to diocotron perturbations, despite being mildly hollow. On longer plasma columns, diocotron instabilities (which are isomorphic to the Kelvin-Helmholtz instabilities for fluids) cause an initially hollow column to rapidly ($\tau \sim 1$ msec) "collapse" radially inward to a monotonically decreasing density and rotation profile. It is an empirical observation, that as the plasma is made shorter and shorter, hollower and hollower columns are found to be stable with respect to diocotron instabilities. This theoretically unexplained stabilization is yet another example of how finite length effects alter the behaviour of the plasma.

The equilibration rate, $\nu_{eq}$, was defined by Driscoll to be the exponential
Figure 4.5: Previous measurements of equilibrium rate, $\nu_{eq}$, versus magnetic field, $B$. Transport to thermal equilibrium was observed and measured by Driscoll et al. using short, hollow plasmas. The rate was found to be as much as 4000 times larger than Classical theory predictions and to scale as $\nu_{eq} \propto B^{-1}$. 
rate with which the density profile approaches thermal equilibrium. The “distance” from equilibrium was calculated from the radial integral of the difference between the density profiles at any given time and the final time. (Note: in Reference [16] the transport is discussed in terms of the time $\tau_{eq}$, whereas, I choose to present the inverse of this quantity $\nu_{eq} \equiv \tau_{eq}^{-1}$. In addition, I display the $z$-integrated density in Figure 4.5, instead of the density at $z = 0$ as displayed in Reference [16].)

The equilibration rate is shown in Figure 4.5 to approximately follow the scaling,

$$\nu_{eq} = 0.5 \text{ sec}^{-1} \left[ \frac{B}{100 \text{G}} \right]^{-1},$$  \hspace{1cm} (4.23)

over the range in magnetic field $B = 47 - 470 \text{ G}$. The data clearly shows that transport due to classical velocity-scattering collisions does not describe the dominant transport occurring in these plasmas. The measured rates are up to 4000 times larger than those predicted by *Classical* theory, and they scale differently with magnetic field.

This magnetic field scaling also does not agree with the 3D *Long-Range* theory, which predicts transport rates that scale as $B^{-2}$. The observation that the measured transport rate scales as $B^{-1}$ provided the impetus for the “hollow only” 2D theory. However, a detailed comparison to this theory was not made, and as the authors state in Reference [16], “the present data are not sufficiently detailed to verify any one transport mechanism.” The transport rate is a global quantity that ignores local variations in the transport, and thus detailed comparisons to local theories are not possible.

An earlier data set, which appeared in two conferences proceedings [24, 54], should be disregarded. This data is extremely crude and has a considerable amount of error associated with it. These reported equilibrium times are only “eye-ball” estimates, as opposed to the data presented in this section, which was calculated
from a rigorously defined quantity.

In the following sections, I present measurements of the local kinematic viscosity obtained by measuring the radial flux of particles at all radii. These measurements verify the basic model of local viscous transport, and they provide challenges to the theoretical predictions presented in the previous section.

4.5 Measured Local Quantities

I measure transport toward thermal equilibrium, and determine the local kinematic viscosity, $\kappa_x(r)$, from radial density profiles taken with the EV apparatus. All experimental quantities are calculated from the $z$-integrated radial density profiles at two times $n_c(r, t_1)$ and $n_c(r, t_2)$, and from the perpendicular temperature $T_{\perp}(r)$ at the average time $t = (t_1 + t_2)/2$. Example of typical density and temperature profiles are shown in Figure 4.6, along with the change in density $\Delta n_c(r)/\Delta t \equiv [n_c(r, t_2) - n_c(r, t_1)] / [t_2 - t_1]$. The time, $\Delta t$, between the two shots was kept relatively short, so that the plasma parameters do not change substantially during the evolution.

Even at relatively early times in the plasma evolution, the temperature profile is fairly constant over the diameter of the plasma, so I typically use a single averaged value $T$ for the entire plasma. The density profile, on the other hand, is intentionally created to have radial variations, and is often quite “bumpy”. Particle transport occurs as these bumps in the density profile smooth out over time. Essentially, I calculate the experimental viscosity from the shear in the rotation frequency and from the particle transport associated with the “smoothing” of bumps in the density profile.

To calculate the flux and to solve Poisson’s equation, as discussed in Section 2.4, I fit the density profile data. The data is of sufficient detail that the fits
would be indistinguishable from the raw data curves shown in the lower portion of Figure 4.6. In the top portion of this figure, the fit is shown in relation to the difference in the density profiles $\Delta n_c/\Delta t$.

### 4.5.1 Rotation Rate and Shear

I calculate radially-dependent rotation frequencies, $\omega_{tot}(r)$, $\omega_E(r)$, and $\omega_D(r)$ from the $z$-averages of the quantities $\bar{\omega}_{tot}(r,z)$, etc. These $z$-dependent rotation frequencies are determined using Equation 4.3 with the “Poisson” density $\bar{n}(r,z)$ and potential $\bar{\phi}(r,z)$. 

**Figure 4.6:** Change in density of an initially “bumpy” profile. In the top portion of the figure $\Delta n_c/\Delta t$ calculated from fits to the two density profiles (solid line) is shown to capture the radial variations in the raw data.
The experimental $\mathbf{E} \times \mathbf{B}$ rotation frequency is thus defined to be

$$
\omega_E(r) \equiv \int \frac{dz}{L_c} \hat{n}(r, z) \left[ \frac{c}{Br} \frac{\partial \phi(r, z)}{\partial r} \right],
$$

(4.24)

and the experimental diamagnetic drift is

$$
\omega_D(r) \equiv \int \frac{dz}{L_c} \hat{n}(r, z) \left[ -\frac{cT}{eB} r \frac{1}{r} \frac{\partial \hat{n}(r, z)}{\partial r} \right].
$$

(4.25)

In the equation for $\omega_D(r)$, I pull the temperature out of the derivative to reflect the experimental practice of fitting the radial temperature profile to one value $T$.

To obtain one set of parameter values for comparison to theory, I take the average of the rotation frequencies, plasma density, and plasma length for the two different hold times $t_1$ and $t_2$. The shears in the rotation frequency are then calculated from these $z$-averaged and time-averaged quantities, where the total shear is

$$
\frac{\partial \omega_{\text{tot}}(r)}{\partial r} \equiv \frac{\partial \omega_E(r)}{\partial r} + \frac{\partial \omega_D(r)}{\partial r}.
$$

(4.26)

In Figure 4.7, the rotation and shear profiles are shown for the density profiles and temperature data of Figure 4.6. The $\mathbf{E} \times \mathbf{B}$ rotation frequency decreases monotonically from a peak value at the center of the trap, and the diamagnetic drift is generally much smaller and considerably more bumpy.

The shear in the diamagnetic drift contributes significantly to the total shear, even though the diamagnetic drift itself does not contribute significantly to the total rotation frequency. The contribution of the diamagnetic term to the shear is therefore important in answering the question: Is viscous transport driven by the shear in the total rotation frequency or just the shear in the $\mathbf{E} \times \mathbf{B}$ drift frequency? In Section 4.6 I show that it does indeed appear to be the total shear given by Equation 4.26 that drives viscous transport.
Figure 4.7: Radial dependence of rotation rate and shear for the “bumpy” density profile of Figure 4.6. (a) The shear in the total rotation frequency depends strongly on the shear in the diamagnetic frequency, as well as on the shear in the $\mathbf{E} \times \mathbf{B}$ frequency; even though (b) the total rotation frequency is mostly dominated by the $\mathbf{E} \times \mathbf{B}$ term alone.

4.5.2 Measured Stress and Coefficient of Viscosity

I calculate the experimental stress from the change in the measured density as

\[
P_x(r) \equiv -\frac{cB}{c} \frac{1}{r^2} \int_0^r dr' r' \int_0^{r'} dr'' \frac{r'' \Delta n_e(r'')}{\Delta t} \tag{4.27}
\]

or

\[
P_x(r) \equiv \frac{cB}{c} \frac{1}{r^2} \int_0^r dr' \frac{r'^2}{r'^2} \Gamma_x(r')
\]

This definition is derived by integrating the radial flux given by Equation 4.4.

The stress $P_x$ is a measure of the transport of angular momentum, analo-
hous to the flux $\Gamma_x$ being a measure of the transport of particles. In Equation 2.6, I relate the loss (or gain) of particles to the flux at the trap wall, and here I relate the loss (or gain) of angular momentum to the stress at the wall as,

$$\frac{\partial \mathcal{L}_\theta}{\partial t} = -2\pi R_w^2 L_c \, P_x(R_w). \quad (4.28)$$

This equation is derived from the definition of the stress given by Equation 4.27 and the time derivative of the canonical angular momentum (Equation 2.23).

Since like-particles interactions conserve the total angular momentum, the stress should vanish at the wall ($P_x(R_w) = 0$) for transport due entirely to viscous forces, or due to any other internal interaction. A positive value of the stress at the wall ($P_x(R_w) > 0$) implies a loss of angular momentum from the plasma, such as would be caused by static asymmetries (Chapter 3). For short plasmas, asymmetry transport is minimized, and the total angular momentum is conserved to within the 1% accuracy of the measurement. Conservation of angular momentum is found to hold even in the presence of substantial viscous transport, such as shown in Figure 4.1.

The experimental viscosity is defined locally as the ratio of the local experimental stress to the local shear in the total fluid rotation frequency, as

$$\eta_x(r) \equiv \frac{P_x(r)}{r \, \frac{\partial \omega_{\text{tot}}(r)}{\partial r}}. \quad (4.29)$$

The kinematic viscosity is calculated experimentally as

$$\kappa_x(r) \equiv \frac{\eta_x(r)}{m_c \, n_c(r)} = \frac{P_x(r)}{r \, \frac{\partial \omega_{\text{tot}}(r)}{\partial r}}. \quad (4.30)$$

In the above equation, I refer to the numerator as the scaled stress, and the denominator as the normalized total shear.

In Figure 4.8(a), examples of experimental values for the scaled stress and normalized total shear are plotted as a function of radius. The ratio of these two
Figure 4.8: Radial dependence of experimental stress and measured viscosity. (a) Radial variations in the scaled experimental stress roughly coincide with the radial variations of the normalized total shear. (b) The kinematic viscosity calculated from the ratio of $P_x/r$ to $-n_e m_e \partial \omega_{\text{tot}}/\partial r$. The solid diamonds denote the “first-bump”, where the viscosity data is most accurate.

Curves at each radial position of the data fit is plotted in Figure 4.8(b) as the kinematic viscosity. The error bars of the kinematic viscosity are determined by propagating the shot noise of the raw data through the calculations of both $P_x$ and $\partial \omega_{\text{tot}}/\partial r$. The error is relatively large, and the value of $\kappa_x$ fluctuates substantially, when either the scaled stress or normalized total shear is relatively small. On the other hand, near the peaks in both curves, the calculated error is relatively low, and the measured kinematic viscosity is fairly constant.
Figure 4.9: Experimental stress compared to viscous model. The solid diamonds are for the “first-bump” ($r = 0.12 - 0.60$). (a) The scaled stress, $P_x/r$, is roughly proportional to the normalized total shear, $-n_\text{e} m_\text{e} \partial \omega_{\text{tot}} / \partial r$, with the constant of proportionality being $\kappa_x$. (b) The scaled stress is not proportional to the normalized $E \times B$ shear alone.

4.6 Comparisons to the Viscous Model

4.6.1 Monotonic Profile

The basic viscous model given by Equation 4.2 is a good qualitative match to the data, in that the measured local transport appears to be proportional to the local shear in the total rotation frequency $\partial \omega_{\text{tot}} (r)/ \partial r$. Evidence in support of this statement is shown in Figures 4.8 to 4.10. In Figure 4.8(a), the radial variations in the scaled stress $P_x/r$ can be seen to qualitatively match the radial variations in the
normalized total shear $-m_e n_e \partial \omega_{\text{tot}} / \partial r$. In Figure 4.9(a), the values of these two curves are plotted against each other, and the scaled stress is roughly proportional to the normalized total shear. The constant of proportionality between the two quantities is just the kinematic viscosity $\kappa_x$, which is plotted in Figure 4.8(b) for each grid point. The solid diamond points in both figures correspond to the measured values for the “first bump” (i.e. $r = 0.12 - 0.60$ cm), where I believe the data to be most accurate.

A viscous model in which the stress is driven by shears in the $\mathbf{E} \times \mathbf{B}$ velocity alone is not supported by the data. In Figure 4.8(a), the radial variations in the normalized $\mathbf{E} \times \mathbf{B}$ shear $-m_e n_e \partial \omega_E / \partial r$ do not qualitatively match the radial variations of $P_x / r$, and in Figure 4.9(b) these two quantities are shown to not be proportional. In case after case, the measured transport is qualitatively well described by a viscous model in which transport is driven by the local shear in the total rotation frequency, but not by the local shear in the $\mathbf{E} \times \mathbf{B}$ drift frequency alone.

In a local model of viscous transport, the kinematic viscosity is not necessarily constant across the entire plasma, and can change as the local plasma parameters change. For instance, in Figure 4.8(b), it is not surprising that the measured value of $\kappa_x$ decreases by nearly a factor of 2 from the “first bump” near $r = 0.36$ cm compared to the “second bump” near $r = 1.25$ cm, where the local density, radial position, and shear are all different. While the viscosity coefficient can be different in different regions of the plasma, theoretically, it should always be positive. Therefore, the observation that the scaled stress and normalized total shear both change sign near the same radial position, as shown in Figure 4.8(a), is another indication that the two quantities are well correlated.

Further evidence that the viscous model of Equation 4.2 is an adequate
Figure 4.10: Experimental flux compared to viscous model. (a) The radial variations in the measured flux agrees with a viscous model and not with a diffusive model. (b) Measured flux as a function of viscous model parameters. (c) Measured flux as a function of diffusive model parameters.
description of the data is shown in Figure 4.10. In Figure 4.10(a) the measured particle flux calculated from the change in density shown in Figure 4.6 agrees with the predicted viscous flux, but not with a simple diffusive model.

In Figure 4.10(b), the measured flux is shown to be well correlated with the radially dependent quantities of a simple viscous model. The x-axis in this figure is the radially dependent quantities of the theoretical viscous flux (Equation 4.4) with \( \kappa \) taken to be a constant, i.e. \( \eta(r) \propto n(r) \). On the other hand, in Figure 4.10(c), the measured flux is shown to not be well correlated with a simple diffusive model, where the flux is proportional to the normalized density gradient, \( \Gamma_x \propto n_e \partial n_p/\partial r \).

Note: The dashed lines in parts (b) and (c) of Figure 4.10 are arbitrarily drawn straight lines which pass through the origin, and are intended to demonstrate how well the \( x \) and \( y \) axes of each graph are correlated with each other.

The transport measurements presented in this chapter also do not agree with asymmetry-induced transport. While an an asymmetry with an \( m = 1 \) azimuthal variation is found to smooth out density profiles, this transport is characterized by a loss of angular momentum (quantified in Chapter 3 by \( \nu_{(r^2)} \)) and by an exclusively outward flux of particles and angular momentum. The measured “viscous” flux of this chapter, on the other hand, shows substantial inward transport of particles and momentum, and occurs even when angular momentum is conserved. In addition, the viscous transport is shown to be roughly proportional to the shear in the rotation frequency, whereas external asymmetries induce transport even on shear-free plasmas with flat (“top-hat”) density profiles. Furthermore, viscous transport scales differently with magnetic field and length than asymmetry-induced transport, as shown in the following section.

Taken together, all the experimental evidence indicates that the transport to thermal equilibrium is well described by viscous interactions proportional to the
Figure 4.11: Radial dependence of the experimental stress and measured viscosity for a hollow plasma. The radial dependence of the scaled stress, $P_x/r$, matches the normalized total shear, $-n_e m_e \partial \omega_{tot}/\partial r$ in agreement with a local model of viscous transport.

shear in the total rotation frequency.

4.6.2 Hollow Profile

The local model of viscous transport applies equally well to plasmas with either hollow or monotonic density and/or rotation profiles. For example, in Figure 4.11(a) the radial variations in the scaled stress are shown to coincide with the radial variations in the normalized total shear for a plasma having a hollow density and rotation profiles. The experimental quantities were calculated from density profiles taken from the Driscoll-hollow data set described in Section 4.4.
Specifically, I used profiles at $t_1 = 0.1$ sec (such as shown in Figure 4.5) and
$t_2 = 0.4$ sec (not shown). It appears that even in the case of a hollow plasma, local
viscous forces drive the plasma transport.

The magnitude of the measured kinematic viscosity is also observed to be
roughly the same for hollow or monotonic profiles having the same plasma param-
eters. In Figure 4.11(b), the kinematic viscosity for the hollow profile is shown to be
about $\kappa \approx 9$ cm$^2$/s near the negative peak in the scaled stress, where the estimated
error is smallest. In the next section, this value is shown in Figure 4.12 to be in
good agreement with kinematic viscosity measurements from peaked monotonic
profiles at the same magnetic field ($B = 190$ Gauss).

4.7 Magnetic Field and Length Dependence

I find that the kinematic viscosity increases with magnetic field and de-
creases with plasma length approximately as

- $\kappa_x \propto B/L_p$.

In terms of the rigidity, I find that the viscosity is greater for more rigid plasmas
approximately as $\kappa_x \propto R$. This scaling suggests an enhancement of the viscosity
due to multiple interactions in a finite length plasma. In Section 4.9, I show that
this enhancement can be described by an increase in the number of collisions,
$N_{coll} \propto B/L_p$, of interacting electrons.

To obtain experimental scalings, I calculate the error-weighted average of
the kinematic viscosity for a specific radial region in the plasma. Each average
viscosity value is calculated from the transport measurements between two density
profiles. I only display averages for which the relative error, $\delta \kappa_x/\kappa_x$, is less than
1. As an example: The solid diamond points of Figure 4.8(b) are for the range in
radius $r = 0.12 - 0.60$ cm. The weighted average for these 9 points is $\kappa_x = 3.5 \text{ cm}^2 / \text{s}$ with a relative error of $\delta \kappa_x / \kappa_x = 0.03$, and thus this point is displayed in the following data plots.

Most of the data in this chapter is for the “first-bump” in the measured stress, which is the radial region

- $r = 0.12 - 0.60$ cm $\equiv$ first-bump.

The average kinematic viscosity in this region corresponds to the transport of the central density. I focus on this region of the plasma, because the measurements of viscosity near the center of the plasma are more accurate than those near the edge. Three effects contribute to this: (1) Transport due to inherent trap asymmetries is weak near the center of the plasma, but complicates the local transport measurements at larger radii. In Section 3.8.1, I showed that the transport due to inherent trap asymmetries varies roughly as $\Gamma_x(r) \propto r^4$. (2) Both the measured stress and the total shear are relatively large near the center of the plasma, giving a small relative error, as shown in Figure 4.8(b). (3) The integral nature of the quantity $P_x(r)$, means that unknown systematic errors (such as small differences in collection efficiency) accumulate with radius.

The data in this chapter fall within the following parameter range:

- $B = 95 - 470$ Gauss
- $L_p = 4 - 24$ cm
- $n_p = (0.5 - 1.5) \times 10^7 \text{ cm}^{-3} \approx 10^7 \text{ cm}^{-3}$
- $T = 0.5 - 2.0 \text{ eV} \approx 1 \text{ eV}$.

The density and temperature ranges mark the extreme values of the data, but the vast majority is within 30% of the approximate values listed.
Figure 4.12: Kinematic viscosity, \( \kappa \), versus magnetic field, \( B \). The measured kinematic viscosity increases with magnetic field strength roughly as \( B^1 \), and is as much as \( 10^4 \) times larger than predictions from Classical theory. There is no substantial difference between measurements from plasmas with either monotonic rotation profiles (open diamonds) or hollow rotation profiles (solid squares).

4.7.1 Dependence on Magnetic Field: \( \kappa_x \propto B^1 \)

Measurements of the kinematic viscosity are found to be orders of magnitude larger than predictions from Classical theory and to increase with magnetic field strength roughly as \( \kappa_x \propto B^1 \). In addition, there appears to be no substantial difference between the measured viscosity for a hollow profile and that for a monotonic profile.

In Figure 4.12, I plot measured values of viscosity corresponding to both monotonic and hollow rotation profiles. The open diamonds represent the weighted average of \( \kappa_x(r) \) near the center \( (r = 0.12 - 0.60 \, \text{cm}) \) of monotonic plasmas, e.g.
the average of the solid diamond points in Figure 4.8(b). The solid square points represent the weighted average of $\kappa_c(r)$ near the edge ($r = 1.0 - 1.8$ cm) of hollow plasmas, *e.g.* the average of the solid squares in Figure 4.11(b). The kinematic viscosity values for the hollow profiles are determined from the Driscoll-hollow data set discussed in Section 4.4. (Note: Only a small fraction of the Driscoll-hollow data was taken in a manner appropriate for calculating experimental values of the stress and flux.)

The data in Figure 4.12 is for short plasmas, with the confinement length kept fixed at $L_c = 7.89$ cm for both the monotonic and hollow plasmas. In addition, the calculated plasma length, $z$-averaged density, temperature, and scaled $\mathbf{E} \times \mathbf{B}$ shear are about the same for all the data at $L_p \approx 4.3$ cm, $n_p \approx 10^7$ cm$^{-3}$, $T \approx 1$ eV, and $\partial \omega_E/\partial r \times [B/100 \text{ G}] \approx 2 \times 10^6$ sec$^{-1}$ cm$^{-1}$.

I find that there is no substantial difference in the magnitude of the measured values of kinematic viscosity for monotonic as compared to hollow rotation profiles. The similarity between the measured values occurs despite the fact that for monotonic profiles the viscosity is measured for a radially outward transport of particles and angular momentum, whereas in the case of hollow profiles the transport is radially inward. Recall from the previous section, that in both cases the measured stress is proportional to the local shear in the total rotation frequency.

In Figure 4.12, *Classical* theory predictions for the kinematic viscosity are shown to be much smaller and scale differently with magnetic field than the measured values. These predictions were calculated from the formula for $\kappa_{cl}$ (Equation 4.11) using the average experimental values for $n_p$ and $T$. At the highest field, the disagreement between $\kappa_{cl}$ and $\kappa_c$ is more than 4 orders of magnitude. Driscoll previously observed that a mechanism other than classical velocity-scattering collisions is responsible for global transport towards thermal equilibrium. Here, I
confirm this observation with measurements of the local coefficient of viscosity.

In Figure 4.12, the kinematic viscosity increases by about a factor of 5 when the magnetic field is increased by a factor of 5. While there is a spread in the data (about a factor of 3), the dashed line shows that the kinematic viscosity scales roughly as $\kappa \propto B^1$. The magnetic field scaling for hollow profiles is not different than the scaling for monotonic profiles.

Predictions of the 3D Long-Range theory, while much closer than Classical theory, are smaller than the measured values and also scale differently with magnetic field. These predictions were calculated from the formula for $\kappa_{3D}$ (Equation 4.13) using the above experimental parameter values. For this data, the interaction is predicted to be limited by velocity diffusion, since $\tau_{\text{diff}} \approx (0.1 - 0.5) \times \tau_{\text{shear}}$. In Figure 4.12, the 3D theory is shown to be about a factor of 3 smaller than the measurements at low field and about a factor of 20 smaller at the largest fields available on EV. Note that an enhancement of the 3D theory due to velocity caging would only account for a factor of 3 at all magnetic fields, and therefore, does not accurately describe the data at the higher magnetic fields. If the transport is indeed due to long-range $\mathbf{E} \times \mathbf{B}$ drift collisions, the disagreement between the 3D theory and the measurements is indicative of a 2D enhancement as described in Section 4.3.2.

The nominal 2D Long-Range drift theory predicts the observed $B^1$ scaling of the measured viscosity coefficient, but at about a factor of 10 larger in magnitude. The comparison is made between values of $\kappa_{2D}$ (Equation 4.20) using the parameter values listed above, along with the experimental averages: $r = 0.36$ cm and $\partial L_p/\partial r = 0.15$. The parameter values imply a 2D interaction distance of $\delta_{2D} = 0.19$ cm, which is on the order of the Debye length, $\lambda_D \approx 0.24$ cm.

The calculation of $\kappa_{2D}$ assumes that $\delta_{2D} \ll \text{lambda}_D$ and $\delta_{2D} \ll r$. Since
Figure 4.13: Kinematic viscosity versus plasma length. The measured kinematic viscosity is scaled by the magnetic field. A $L_p^{-1}$ scaling is drawn for comparison.

these assumptions are not strictly satisfied, there is substantial question as to the applicability of the calculated 2D predictions to the data in Figure 4.12. In Section 4.8, I present a slightly better comparison test of the 2D Long-Range theory.

The 2D theoretical enhancement which applies only to hollow profiles is not supported by the data, since both hollow and monotonic profiles exhibit approximately the same degree of enhancement. In fact, it was the measurements displayed in Figure 4.12 that prompted the analysis that led to the more recent, nominal 2D Long-Range theory, which applies to both hollow and monotonic profiles.
4.7.2 Length Dependence: $\kappa_x \propto L_p^{-1}$

In Figure 4.13, the kinematic viscosity is shown to decrease with increasing plasma length. For the data presented in this figure, I scale out the assumed $\kappa \propto B^1$ magnetic field dependence, and use the same averaging process as for the data of Figure 4.12. The data spans the range in confinement length $L_c = 7.89 - 27.73$, which gives a range in calculated plasma length of $L_p \approx 4.3 - 24.3$ cm. In Figure 4.13, I only display data for monotonic profiles, since hollow profiles are not stable at the longer lengths.

The kinematic viscosity is shown to decrease by about a factor of 5 as the plasma length increases by about a factor of 6. While there is considerable spread in the data, a dashed line is drawn in Figure 4.13 to suggest a possible $\kappa \propto L_p^{-1}$ scaling. The observed decrease in viscosity with length indicates that the finite size of the plasma affects the rate at which electrons interact with each other. Therefore, theories of viscous transport that ignore the finite length of the plasma, such as the 3D Long-Range theory, do not accurately describe the data.

4.7.3 Rigidity Dependence

In Figure 4.14, the entire range of kinematic viscosity data is compared to the 3D Long-Range theory. For this figure, I scale the kinematic viscosity by the product of the simple collision frequency and the Debye length squared. This product only depends on plasma parameters as $\nu_c \lambda_D^2 \propto T^{1/2}$, and is fairly constant for the entire range of the data at $\nu_c \lambda_D^2 \approx 0.5 \text{ cm}^2/\text{s}$.

The scaled viscosity is shown to increase as a function of the local rigidity $R = \bar{f}_b/f_E \propto B/L$, where the proportionality reflects the fact that the density and temperature do not vary substantially in this data set.

The measured viscosity is found to be larger than the 3D Long-Range theory
Figure 4.14: Kinematic viscosity versus rigidity. The measured kinematic viscosity is larger than the 3D Long-Range theory prediction, with an enhancement that increases with rigidity $\mathcal{R}$.

Predictions, with greater discrepancy at larger rigidity. Near a rigidity of 1, the 3D theory is in rough agreement with the measurements, but for $\mathcal{R} \approx 10$ the 3D theory is about a factor of 20 less than the measured kinematic viscosity. A dashed line shows a $\kappa \propto \mathcal{R}^3$ scaling for comparison. Note that the increase in the measured viscosity with rigidity is in sharp contrast to the decrease in asymmetry-induced transport with rigidity, as shown in Chapter 3 for slightly rigid plasmas (i.e. $\langle \mathcal{R} \rangle = 1 \text{ to } 10$).
\[ R \equiv f_b / f_E \]

**Figure 4.15:** Measured viscosity compared to predictions from 2D *Long-Range* theory. The predicted enhancement from 2D *Long-Range* theory is about a factor of 10 larger than the measurements. The data spans a range in local rigidity of about \( R = 1 - 10 \).

### 4.8 Comparison to 2D *Long-Range* Theory

In this section, I compare the measured kinematic viscosity to predictions of 2D *Long-Range* theory. The 2D prediction, \( \kappa_{2D} \) (Equation 4.20) is intended to be an enhancement to the 3D viscosity \( \kappa_{3D} \) (Equation 4.13). Therefore, in comparing the measurements to the theory, I subtract off the 3D theory predictions from the data and scale this difference by the 2D prediction as \( (\kappa_x - \kappa_{3D}) / \kappa_{2D} \). The resulting quantity is plotted as a function of the local rigidity in Figure 4.15 to show the range of the data.

The predicted enhancement from the 2D theory does not agree with the
measured data. In Figure 4.15, the predicted enhancement is about a factor of 10 larger than the measured value over the entire range in the experimental rigidity $0.8 < \mathcal{R} < 10$. Here, in an attempt to satisfy the inequalities in Equation 4.21, I only plot data for which $\delta_{2D} < \lambda_D$ and $\delta_{2D} < 0.5 \tau$.

4.9 Dependence on the Number of Collisions

In this section, I present a simple empirical formula that describes the measured values of the kinematic viscosity. This formula incorporates the interaction distance of the 3D theory, $\delta = \lambda_D$, and an effective collision frequency, $\nu_{eff} = (1 + N_{coll}) \nu_c$, that is inspired by the 2D theory.

Here $N_{coll}$ is the average number of additional collisions between a pair of interacting electrons defined as

$$N_{coll} \equiv \frac{\overline{F}_b}{|r \partial \omega_E / \partial r|}.$$  (4.31)

The electrons interact multiple times as they bounce back and forth, until they separate azimuthally due to rotational shear in the plasma; this is similar to the shear-separation described Equation 4.15 for the 3D Long-Range theory except there the factor appears in a logarithmic term. The above equation for $N_{coll}$ becomes unphysical near the center of the plasma ($r = 0$) and at points for which the $\mathbf{E} \times \mathbf{B}$ shear is zero; in this case another effect (such as velocity scattering collisions) will presumably limit the interactions. For Figure 4.16, I calculate $N_{coll}$ for each set of profiles using the average values of $\overline{F}_b$, $r$, and $|\partial \omega_E / \partial r|$ in the same radial region ($r = 0.12 - 0.60$ cm) where each average value of $\kappa_x$ is calculated.

In Figure 4.16, the scaled kinematic viscosity is shown to increase with the number of collisions, $N_{coll}$. The prediction from the 3D Long-Range theory is in agreement with the measured data for $N_{coll} \approx 1$, where the enhancement due to
Figure 4.16: Kinematic viscosity versus the average number of collisions, $N_{\text{coll}}$. The measured kinematic viscosity is larger than the 3D Long-Range theory prediction, with an enhancement that increases with the average number of collisions, $N_{\text{coll}}$, between interacting electrons.

multiple collisions is weak. At larger values of $N_{\text{coll}}$, the viscosity is larger than the 3D predictions, by an amount that is approximately proportional to $N_{\text{coll}}$. Motivated by this data, I suggest the following simple empirical formula for viscosity in a pure-electron plasma

$$\kappa_{\text{em}} = (1 + N_{\text{coll}}) \nu_c \lambda_D^2.$$  \hspace{1cm} (4.32)

4.10 Discussion & Directions for Future Work

The measurements on pure-electron plasmas presented in this chapter verify that like-particle transport toward thermal equilibrium is well described by a vis-
Viscous fluid model for both hollow and monotonic plasmas. Experimental values for the coefficient of viscosity are found to agree much more closely with Long-Range theories than with Classical theory. The measured values show an enhancement over 3D Long-Range predictions that scales roughly as $B^3$ and $L_p^{-1}$, but is smaller than the enhancement predicted by the 2D Long-Range theory. A simplified hybrid of the 3D and 2D Long-Range theories works well at describing the data. Further theoretical and experimental work is required to quantitatively understand the observed finite-length enhancement.

The experiments on viscous transport were conducted exclusively in the so-called slightly-rigid regime, where the rigidity is in the approximate range $1 \lesssim \mathcal{R} \lesssim 10$. The 3D Long-Range theory ignores finite length effects, and is perhaps more appropriately applied to a floppy plasma with $\mathcal{R} < 1$. In fact, the measured viscosity is found to agree with the 3D Long-Range theory at the lowest experimental rigidity, $\mathcal{R} \approx 1$. One direction that future work could take would be to measure the viscosity of floppy plasmas. Such measurements could be incisive in determining the validity of the 3D Long-Range theory.

Measurements of viscosity are also needed in the highly-rigid regime, where $\mathcal{R} > 10$. In Chapter 3, asymmetry-induced transport is shown to exhibit a dramatic change in transport scalings when the average rigidity is increased above $\langle \mathcal{R} \rangle = 10$. It is not known whether viscous transport exhibits a similar change. In addition, measurements in the highly-rigid regime will also allow for a better comparison between experiments and the 2D Long-Range theory. This comparison would also be aided by a re-working of the theory using less stringent assumptions that are more appropriate to realistic experimental conditions.
4.11 Viscous & Asymmetry-Induced Transport

In this section, I compare predictions from the empirical formulas for both viscous and asymmetry-induced transport to the radial particle flux in a plasma with high shear and low rigidity. Viscous effects dominate the transport near the center of the plasma, whereas inherent trap asymmetries cause transport near the edge of the plasma. The sum of the predictions from the two empirical formulas does quite well at describing the measured flux across the entire radius of the plasma.

In Figure 4.17, the measured flux, $\Gamma_x(r)$, is shown for a plasma with a relatively low rigidity ($\mathcal{R} = 1.5$); here, the magnetic field is $B = 95$ Gauss and the confinement length is $L_c = 15.78$ cm ($L_p(0) \approx 12$ cm). The density profiles for this data are similar to the “bumpy” profiles shown in Figure 4.6 for a shorter plasma ($L_p(0) \approx 4$ cm) with a higher rigidity $\langle \mathcal{R} \rangle = 4.3$. The measured flux shown here is substantially larger at the edge of the plasma than is the measured flux for the shorter plasma (shown in Figure 4.10(a)); this is because inherent trap asymmetries cause transport primarily at the edge of the plasma, and this asymmetry-induced transport is greater for longer, less rigid plasmas.

An empirical form for the flux due to trap asymmetries is calculated from Equation 3.21 using $m = 4$ as

$$
\Gamma_{trap}(r) = \alpha V_t n_c(r) A^2 \mathcal{R}^{-2}(r) \left( \frac{r}{R_w} \right)^4,
$$

where $\alpha$ is given by Equation 3.22. The value for $V_t$ is determined from the measured expansion rate, $\nu_{bg} = 0.11$ sec$^{-1}$, and the formula $\nu_{bg} = 7 V_t \langle \mathcal{R} \rangle^{-2}$ (see Section 3.8.1). This gives $V_t = 0.036$ Volt $= 1.2 \times 10^{-4}$ Statvolt, which is reasonably close to the average effective trap asymmetry of 0.06 Volt (Table 3.1). (For this experiment the confinement region does not include the S6 sectored ring.)
Figure 4.17: Flux due to both viscous and inherent trap asymmetries. (a) Measured flux $\Gamma_x(r)$ compared to empirical models for transport due to trap asymmetries $\Gamma_{\text{trap}}(r)$ and viscous-driven interactions $\Gamma_{\text{vis}}(r)$. (b) Measured flux compared to the sum of the two models.
The choice of $m = 4$ for the azimuthal mode number of the inherent asymmetry is somewhat arbitrary, but this choice is supported by the observed radial dependence of the transport on a flat profile as shown in Figures 3.19(a), 3.21, and 3.39.

An empirical form for the viscous flux, $\Gamma_{\text{vis}}(r)$, is calculated using the empirical kinematic viscosity in Equation 4.32, along with the formula for the local viscous flux given by Equation 4.4 with no adjustable parameters. The quantities $\nu_c(r), \lambda_D(r)$, and $N_{\text{coll}}(r)$ are radially dependent, and are calculated from the local values of $n_p(r), L_p(r)$, and $\partial \omega_E(r)/\partial r$. To avoid exceeding large values of $N_{\text{coll}}$ near the center of the plasma, I replace $r$ with $\lambda_D$ in Equation 4.31 for the few points where $r < \lambda_D \approx 0.24$ cm.
Appendix A

Fourier Components of Applied Asymmetries

In this appendix, I list calculated Fourier components for voltages applied to patches on the sectored rings. These voltages are the applied asymmetries discussed in Chapter 3.

The vacuum potential of the asymmetry is decomposed into azimuthal Fourier components as

$$\Phi_a(r, \theta) = \sum_m \Phi_{a,m} \left( \frac{r}{R_w} \right)^m e^{im\theta}. \quad (A.1)$$

The boundary condition is the azimuthally dependent potential at the wall, which is related to the Fourier components as

$$\Phi_a(R_w, \theta) = V_a \left[ \sum_m A_m \cos(m\theta) + B_m \sin(m\theta) \right]. \quad (A.2)$$

($V_a$ is the same quantity used throughout Chapter 3 to describe the perturbation strength.) Equation A.2 is easily solved for the values of $A_m$ and $B_m$ as

$$A_m = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, \cos(m\theta) \frac{\Phi_a(R_w, \theta)}{V_a} \quad (A.3)$$

$$B_m = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, \sin(m\theta) \frac{\Phi_a(R_w, \theta)}{V_a}. \quad (A.4)$$

For all the example perturbations, I specifically set the point on the sectored ring where $\theta = 0$ so that either all $A_m$’s or alternatively all $B_m$’s are zero.
As an example, consider the nominal $m = 1$ perturbation shown in Figures 3.5 and A.1. For this asymmetry, I set $\theta = 0$ at the center of the sector with the $+V_a$ applied voltage, and the boundary conditions become:

$$
\Phi_a(R_w, \theta)/V_a = \begin{cases} 
1 & : \theta = -\pi/6 \text{ to } \pi/6 \\
-1 & : \theta = 5\pi/6 \text{ to } 7\pi/6 \\
0 & : \text{all other } \theta.
\end{cases}
$$

The solution to Equation A.3 is simply $A_m = [\sin(m\pi/6) + \sin(m\pi/6)]/\pi$ and $B_m = 0$.

In Figures A.1 and A.2, I display schematics and list the relative Fourier strengths for asymmetries applied to either the EV (S6) or CamV (S4) 4-sector cylinders. Both of these cylinders have $60^\circ = \pi/3$ radian sectors separated by $30^\circ = \pi/6$ radian sections of the cylinder frame. The frame is grounded in all examples except for the “$m = 4$ (alternative)”, where the frame is set to $-V_a$.

In Figure A.3, I display schematics and list the relative Fourier strengths for two different asymmetries applied to the CamV (S7) 8-sector cylinder. This cylinder has $25^\circ = 5\pi/36$ radian sectors separated by $20^\circ = \pi/9$ radian sections of the cylinder frame. The frame is grounded for both perturbations. Asymmetric voltages were only applied to this 8-sector cylinder for the experiments presented in Section 3.7.3 concerning two asymmetries applied at opposite ends of the plasma.
\[ m=1 \quad m=2 \quad m=4 \]

\[
\begin{array}{|c|c|}
\hline
 m & A_m \\
\hline
0 & - \\
1 & 0.637 \\
2 & - \\
3 & 0.424 \\
4 & - \\
5 & 0.127 \\
6 & - \\
7 & -0.091 \\
8 & - \\
9 & -0.141 \\
10 & - \\
11 & -0.058 \\
12 & - \\
13 & 0.049 \\
14 & - \\
15 & 0.085 \\
16 & - \\
17 & 0.037 \\
18 & - \\
19 & -0.034 \\
20 & - \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
 m & A_m \\
\hline
0 & - \\
1 & 0.1.103 \\
2 & - \\
3 & - \\
4 & 1.103 \\
5 & - \\
6 & - \\
7 & - \\
8 & -0.551 \\
9 & - \\
10 & -0.221 \\
11 & - \\
12 & - \\
13 & - \\
14 & 0.158 \\
15 & - \\
16 & - \\
17 & 0.276 \\
18 & - \\
19 & - \\
20 & -0.221 \\
\hline
\end{array}
\]

**Figure A.1**: Fourier decomposition of \( m = 1, m = 2, \) and \( m = 4 \) asymmetries on the \((60^\circ)\) 4-sector cylinders. The frame of the ring is grounded in each case.
**one-sector**  \( m=4\) (alternative)  \( m=1\) (clam-shell)

![Diagram](image)

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<th>(m)</th>
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<th>(m)</th>
<th>(A_m)</th>
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**Figure A.2:** Fourier decomposition of one-sector, \( m = 4 \) (alternative), and \( m = 1 \) (clam-shell) asymmetries on the (60°) 4-sector cylinders. The frame of the ring is grounded for the one-sector and \( m = 1 \) (clam-shell), but is set to \(-V_a\) for the \( m = 4 \)-(alternative). The \( m = 1 \) (clam-shell) was not used for the experiments in this thesis.
**Figure A.3**: Fourier decomposition of $m = 1\pm$ and $m = 1 \pm \pm$ asymmetries on the (25°) 8-sector cylinder. In both cases the frame of the cylinder is grounded.
Appendix B
Symbols and Notations

This appendix lists symbols and notations commonly used in this thesis. Whenever necessary, the definition of a symbol or equation number where it is first introduced is given. All equations use the cgs convention.

********** Notation **********

(r, θ, z) Cylindrical coordinate system centered on the trap axis.

(ρ, ψ, z) Cylindrical coordinate system centered on the plasma axis.

y^{3D}(r, θ, z) Three-Dimensional quantity

\ddot{y}(r, z) Quantity with an axial dependence.

y(r) Radially dependent quantity.

y(r) Eq. 2.9 Radial quantity, as calculated from a density-weighted z-average.

\langle y \rangle Eq. 2.12 Density-weighted radial average
******* Fundamental Quantities *******

$-e$  
Electron charge

$m_e$  
Electron mass

$c$  
Speed of light in vacuum

$B$  
Axial magnetic field

$\phi$  
Electrostatic potential

$E$  
Electric field

$\hat{n}(r, z)$  
Eq. 2.7  
Density calculated using Poisson's Eqn.

$n_p(r)$  
Eq. 2.10  
Plasma density

$\langle n_p \rangle$  
Radially Averaged density

$n_c(r)$  
Eq. 2.2  
Measured Confinement Density

******* Lengths *******

$L_c$  
Figure 2.6  
Confinement Length of grounded confinement region

$L_p(r)$  
$L_c \left[ n_e(r)/n_p(r) \right]$  
Calculated Plasma length

$\langle L_p \rangle$  
Radially averaged plasma length

$R_p$  
$(2 \langle r^2 \rangle)^{1/2}$  
Plasma radius

$\tau_e$  
$\dot{\theta}/\Omega_c$  
Thermal electron cyclotron radius

$b$  
$e^2/T$  
Distance of closest approach

$\lambda_D$  
$\sqrt{T/4\pi e^2 n_p}$  
Debye length

$R_w$  
3.81 cm, 3.5 cm  
Wall radius for EV, CamV

$d$  
D/Rw  
Scaled Displacement of plasma from the trap axis

$\Delta$  
Eq. 3.17  
Effective shift in equilibrium

$\delta$  
Sec. 4.2  
Viscous interaction distance between electrons

$\delta_{2D}$  
Eq. 4.19  
Interaction distance in 2D Long-Range Theory
********** Velocities, Rates, and Times **********

\( v_|| \)  
Electron velocity parallel to magnetic field

\( \bar{v} = \sqrt{kT/m_e} \)  
Thermal electron velocity

\( \Omega_e = eB/m_ec \)  
Electron cyclotron frequency

\( \omega_p = \sqrt{4\pi ne^2/m_e} \)  
Electron plasma frequency

\( f_b = v_||/2L_p \)  
Axial bounce frequency

\( \bar{f}_b = \bar{v}/2L_p \)  
Axial bounce frequency of thermal electron.

\( f_E = -E_r c/2\pi rB \)  
\( \mathbf{E} \times \mathbf{B} \) rotation frequency

\( f_{diox} \approx (R_p/R_w)^2 f_E \)  
\( m = 1 \) diocotron mode frequency

\( \omega_D \)  
Eq. 4.25  
Diamagnetic (or pressure) drift

\( \omega_{tot} = \omega_E + \omega_D \)  
Total fluid rotation frequency

\( \nu_{ee} \)  
Eq. 2.20  
Electron-electron collision rate

\( \nu_c = n_p \bar{v} b^2 \)  
Simplified collision frequency

\( \nu_{eff} \)  
Effective Collision Frequency

\( t_{hold} \)  
Plasma Hold time

\( t_{pert} \)  
Duration asymmetric perturbation is applied

Note that \( \omega = 2\pi f \) for all frequencies.
********** Measured Transport Quantities **********

\[ \nu(r^2) = \frac{\langle r^2 \rangle}{d \langle r^2 \rangle / dt} \]  
Expansion rate calculated from the center of the trap

\[ \nu(p^2) = \frac{\langle p^2 \rangle}{d \langle p^2 \rangle / dt} \]  
Expansion rate calculated from the center-of-mass

\[ \nu_{bg} \]  
Background expansion rate due to inherent trap asymmetries

\[ \Delta \nu(r^2) = \nu(r^2) - \nu_{bg} \]  
Net expansion rate

\[ \nu_{n0} = n_e^{-1}(0) \frac{dn_e(0)}{dt} \]  
Rate of change of the central density

\[ \tau_m \]  
Eq. 3.26  
Time for central density to decrease by 1/2

\[ \Gamma_x(r) \]  
Eq. 2.5  
Local radial flux of particles.

\[ P_x(r) \]  
Eq. 4.27  
Experimental stress

\[ \eta_x(r) = -P_x(r)/r \partial \omega_{tot}(r)/\partial r \]  
Experimentally determined coefficient of viscosity.

\[ \kappa_x(r) = \eta_x(r)/m_e n_e(r) \]  
Experimental kinematic viscosity

\[ \nu_{eq} = 1/\tau_{eq} \]  
Rate at which plasma relaxes toward thermal equilibrium

********** Miscellaneous **********

\[ \langle r^2 \rangle \]  
Eq. 3.2  
Mean square radius calculated from the center of the trap

\[ \langle p^2 \rangle = \langle r^2 \rangle - D^2 \]  
Mean square radius calculated from the center-of-mass.

\[ L_\theta \]  
Eq. 2.23  
Total canonical angular momentum

\[ R \]  
\( \bar{f}_b/f_E \), Eq. 2.19  
Plasma Rigidity

\[ \langle R \rangle = \langle \bar{f}_b/f_E \rangle, \text{Eq. 3.5} \]  
Radially averaged Plasma Rigidity

\[ R_0 = \bar{f}_b(0)/f_E(0) \]  
Plasma Rigidity at \( r = 0 \)

\[ V_a \]  
Applied Voltage at the trap wall

\[ \Phi_{a,m} \]  
Azimuthal \( m \)-numbered Fourier component of the applied voltage at the trap wall.

\[ N_{coll} = \frac{\bar{f}_b}{|r \partial \omega_E/\partial r|} \]  
Average number of collisions between bouncing electrons

\[ N_{tot} = \text{Eq. 2.3} \]  
Total number of electrons in the plasma

\[ N_L = N_{tot}/\langle L_p \rangle \]  
Number of electrons per unit length
References


