Electrostatic waves and instabilities in multispecies nonneutral plasmas

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This paper briefly describes waves and instabilities in a multispecies nonneutral plasma that have previously been studied only in neutral plasmas: ion sound waves, drift waves, and ion temperature gradient waves. The theory of their dispersion relations and growth rates is similar to that for neutral plasmas, but results differ in several important respects. For instance, drift waves are not necessarily unstable, the instabilities generally do not cause plasma loss, and they can be controlled (e.g., turned on or off) by manipulation of density and temperature profiles using standard experimental techniques such as centrifugal separation and laser cooling/heating. This should allow precise experimental studies of instability growth and saturation. © *2010 American Institute of Physics*.
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I. INTRODUCTION

In this paper we discuss how several electrostatic waves that have previously been studied only in neutral plasmas can also occur in multispecies nonneutral plasmas, focusing in particular on plasmas for which all species have the same sign of charge. Among these waves are ion sound waves, drift waves, and ion temperature (ITG) gradient waves. The occurrence of these waves does not rely on neutrality of the plasma, but rather on the coexistence of at least two species, at least one of which responds to the wave in a nearly adiabatic fashion, Debye-shielding the disturbance; the others well-approximated by fluid equations. This typically (but not necessarily) requires a large mass ratio between the species, but has nothing to do with the sign of the charge. Thus, nonneutral plasmas can exhibit phenomena of importance in neutral plasma studies such as drift wave or ITG instabilities.

While multispecies dispersion relations for nonneutral plasmas have been derived previously,[1] ion sound waves, drift waves, and ITG waves have not been explicitly considered. We will derive the dispersion relations for these waves in a nonneutral plasma, discussing differences when compared to waves in a neutral plasma, and show that certain drift wave instabilities can be controlled (turned on and off) in such a way as to allow careful studies of their growth and resulting transport.

Early studies of drift waves in quasineutral Q machine plasmas also controlled stability through a combination of stabilizing influences such as ion viscosity and/or Landau damping, and finite Larmor radius effects; and destabilizing influences such as parallel current and centrifugal force due to $E\times B$ rotation.[2–5] More recent studies of ITG modes have observed instability onset through variation of temperature and density gradients.[6] In this paper we argue that multispecies nonneutral plasmas provide another plasma system where such waves can be carefully studied, in a fully trapped plasma (disconnected from sources and sinks at the plasma ends), over a wide range of parameters from low temperature highly collisional (even strongly coupled) plasmas to plasmas with dimensionless parameters similar to those in fusion plasmas. Strong $E\times B$ rotation of nonneutral plasmas also allows study of the effect of $E\times B$ shear on instability and transport.

To illustrate some of these ideas, consider a cylindrical nonneutral plasma column consisting of two species with charge q_α, mass m_α, density $n_\alpha(r)$, and temperature $T_\alpha(r)$, where α is a species label i or e (i.e., ion or electron). Note that e does not necessarily refer to electrons; it really refers to a species with nearly adiabatic response to perturbations. This could be a light ion or a positron rather than an electron. Also all species are assumed to have the same sign of charge, so i and e could refer to an H^- ion and electron, or a Mg^+ ion and a positron, for example.

The unneutralized density gives rise to an equilibrium potential $\phi_0(r)$ that in turn causes a drift rotation of particles at rate $\omega_\alpha(r)$ where

$$\omega_\alpha(r) = \omega_E(r) - \omega_E^2/\Omega_\alpha + O\left(\frac{1}{B^4}\right), \tag{1}$$

$$\omega_E = \frac{c}{Br}\frac{\partial\phi_0}{\partial r} \tag{2}$$

is the $E\times B$ rotation frequency, $\Omega_\alpha = q_\alpha B/m_\alpha c$ is the (signed) cyclotron frequency,[7,8] and finite Larmor-radius (FLR) corrections have been neglected. The second term in Eq. (1) is the polarization drift due to centrifugal force (also sometimes referred to as the centrifugal drift). Equation (1) assumes $\omega_E/\Omega_\alpha \ll 1$, a good approximation in most nonneutral (and magnetic fusion) plasma experiments. The centrifugal drift is usually neglected in studies of waves in nonneutral plasmas, but we will show it can have an important influence on drift wave instability.

Electrostatic waves on the plasma column produce potential perturbations of the form $\delta\phi = \delta\phi(r)e^{ik_z z + i\ell\theta - i\omega t}$. We focus our attention here on waves that satisfy

$$4k_z\bar{v}_i \lesssim \omega - \ell\omega_i \ll k_z\bar{v}_e, \tag{3}$$

where $\bar{v}_\alpha = \sqrt{T_\alpha/m_\alpha}$ is the thermal speed of species α; this is the usual regime of weakly Landau-damped (or growing) low frequency electrostatic waves.[9] However, in nonneutral plasmas the general features of these waves differ in some important respects from their neutral plasma counterparts, so it is worthwhile to re-examine some elements of the basic theory of the waves.

When Eq. (3) is satisfied the perturbed electron density δn_e is nearly adiabatic,

$$\delta n_e = -\frac{q_e \delta\phi}{T_e} n_e (1 + i\beta), \tag{4}$$

where β represents the small out-of-phase nonadiabatic response due to either collisions or electron Landau damping/growth. On the other hand, Eq. (3) implies that ion dynamics can be approximated by fluid theory. The linearized fluid equations are

$$\frac{\partial \delta n_i}{\partial t} + \omega_i \frac{\partial \delta n_i}{\partial \theta} + \delta \mathrm{v}_r \frac{\partial n_i}{\partial r} + n_i \nabla \cdot \delta \mathbf{v} = 0, \tag{5a}$$

$$\frac{\partial \delta \mathbf{v}}{\partial t} + \delta \mathbf{v} \cdot \nabla(\omega_i r \hat{\theta}) + \omega_i \frac{\partial \delta \mathbf{v}}{\partial \theta} = -\frac{q_i}{m_i} \nabla \delta\phi - \frac{1}{m_i n_i} \nabla (n_i \delta T + \delta n_i T_i) + \delta \mathbf{v} \times \hat{z}\Omega_i, \tag{5b}$$

$$\frac{\partial \delta T}{\partial t} + \omega_i \frac{\partial \delta T}{\partial \theta} + \delta \mathrm{v}_r \frac{\partial T_i}{\partial r} + (\gamma - 1) T_i \nabla \cdot \delta \mathbf{v} = 0, \tag{5c}$$

where γ is the ratio of specific heats, and we have neglected ion collisions and assumed an adiabatic equation of state. For our purposes it is sufficient to solve Eq. (5b) for $\delta \mathbf{v}$ in the drift approximation, keeping drift terms only in the cold fluid approximation but keeping warm-fluid corrections to the parallel motion. This results in

$$\delta \mathrm{v}_z = \frac{q_i}{m_i \omega'} k_z \delta\phi + \frac{k_z}{m_i n_i \omega'} (n_i \delta T + T_i \delta n_i), \tag{6}$$

where $\omega' = \omega - \ell \omega_i$ is the Doppler-shifted frequency, and

$$\delta \mathbf{v}_\perp = \frac{c}{B} \left(-\frac{i\ell}{r} \delta\phi \hat{r} + \frac{\partial \delta\phi}{\partial r} \hat{\theta} \right) + \delta \mathbf{v}_p + O\left(\frac{1}{B^3} \right), \tag{7}$$

where the perturbed ion polarization drift $\delta \mathbf{v}_p$ is

$$\delta \mathbf{v}_p = -\frac{c}{B\Omega_i} \left\{ \hat{\theta} \left[\omega' \frac{\ell}{r} \delta\phi + 2\omega_i \frac{\partial \delta\phi}{\partial r} \right] - i\hat{r} \left[\omega' \frac{\partial \delta\phi}{\partial r} + \frac{\ell}{r} \left(2\omega_i + r \frac{\partial \omega_i}{\partial r} \right) \delta\phi \right] \right\}. \tag{8}$$

We will see that in nonneutral plasmas far from the Brillouin limit these polarization terms can be neglected along with other $O(1/B^2)$ terms associated with FLR effects, as opposed to neutral plasmas where such terms can play an important role in drift instability.

Next we solve Eq. (5c) for δT, dropping the polarization terms for simplicity. This results in

$$\delta T = T_i (\gamma - 1) \frac{k_z \delta \mathrm{v}_z}{\omega'} - \frac{\ell c}{B r \omega'} \delta\phi \frac{\partial T_i}{\partial r}. \tag{9}$$

Substituting Eqs. (7)–(9) into Eq. (5a) yields the ion density response

$$\frac{\delta n_i}{n_i} = \frac{q_i k_z^2}{m_i(\omega'^2 - \gamma k_z^2 \bar{\mathrm{v}}_i^2)} \left(1 - \frac{\ell \omega_{T_i}}{\omega'} \right) \delta\phi - \frac{\ell \omega_{D_i}}{\omega'} \frac{q_i \delta\phi}{T_i} + \frac{c}{B\Omega_i n_i} \left[\frac{\partial \delta\phi}{\partial r} + \frac{\ell}{r\omega'} \left(2\omega_i + r \frac{\partial \omega_i}{\partial r} \right) \delta\phi \right] \frac{\partial n_i}{\partial r} + \frac{c}{B\Omega_i} \left[\nabla_\perp^2 \delta\phi + \frac{\ell}{r\omega'} \left(r \frac{\partial^2 \omega_i}{\partial r^2} + 3 \frac{\partial \omega_i}{\partial r} \right) \delta\phi \right], \tag{10}$$

where

$$\omega_{T_\alpha} = \frac{c}{q_\alpha B r} \frac{\partial T_\alpha}{\partial r} \tag{11}$$

and

$$\omega_{D_\alpha} = \frac{c T_\alpha}{q_\alpha B r n_\alpha} \frac{\partial n_\alpha}{\partial r} \tag{12}$$

are gradient (diamagnetic) drift rotation rates.

For simplicity we have dropped warm-fluid corrections to the drift terms in Eq. (10), keeping them to $O(k_z^2 \bar{\mathrm{v}}_i^2 / \omega'^2)$ in the first term describing the density response due to parallel electric fields (so as to capture ion sound wave and ITG dynamics). Even with these simplifications, the $O(1/B^2)$ polarization drift terms are rather complicated, but are dominated by the polarization density term $c/B\Omega_i \nabla_\perp^2 \delta\phi$ when transverse wavelengths are sufficiently short. In what follows we will keep only this term, eventually showing that for nonneutral plasmas far from the Brillouin limit it too can be dropped. Thus, we will see that the only polarization term that is important in such plasmas is the centrifugal drift correction to the equilibrium ion rotation frequency, given in Eq. (1).

Poisson's equation, $\nabla^2 \delta\phi = -4\pi(q_i \delta n_i + q_e \delta n_e)$, closes the system of equations. We first consider the "local" approximation where we replace ∇^2 by $-k^2$ (Ref. 9, p. 425). The resulting dispersion relation is

$$\omega'^3 = c_s^2 k_z^2 (\omega' - \ell \omega_{Ti}^*) - \left(\ell \omega^* + \frac{i\beta\omega'}{1 + k^2 \lambda_e^2 + k_\perp^2 \rho_s^2} \right) \times (\omega'^2 - \gamma k_z^2 \bar{\mathrm{v}}_i^2), \tag{13}$$

where $c_s^2 = c_*^2/(1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2) + \gamma \bar{\mathrm{v}}_i^2$ is the ion sound speed, $c_* = \sqrt{T_e q_i^2 n_i / m_i q_e^2 n_e}$, $\omega_{T_i}^* = \omega_{Ti}/[1 + (\gamma \bar{\mathrm{v}}_i^2 / c_*^2)(1 + k^2 \lambda_e^2 + k_\perp^2 \rho_s^2)]$, $\rho_s = c_*/\Omega_i$, $\lambda_e = \sqrt{T_e / 4\pi q_e^2 n_e}$, and $\omega^* = \omega_{Di} q_i^2 n_i T_e / [q_e^2 n_e T_i (1 + k^2 \lambda_e^2 + k_\perp^2 \rho_s^2)]$ is the drift wave phase speed. The $k^2\lambda_e^2$ term arises from $\nabla^2 \delta\phi$, while the $k_\perp^2 \rho_s^2$ term arises from the polarization density.

Ion sound wave dispersion is obtained by neglecting ω_{Ti}, β, and ω^*, yielding $\omega' = k_z c_s$. Note that c_s is the usual neutral plasma expression for the ion sound speed when $q_i n_i = \pm q_e n_e$, but differs when $q_i n_i \neq \pm q_e n_e$. Condition (3) that the waves are weakly Landau-damped becomes (assuming $k^2 \lambda_e^2 + k_\perp^2 \rho_s^2 \ll 1$),

$$16 \lesssim \frac{T_e q_i^2 n_i}{T_i q_e^2 n_e} + \gamma \ll \frac{T_e m_i}{T_i m_e}. \quad (14)$$

In a neutral plasma with $|q_i|=|q_e|$ and $n_i=n_e$, Eq. (14) can only be satisfied if $T_e \gtrsim (16-\gamma)T_i$. However, in a nonneutral plasma Eq. (14) can be satisfied (and hence weakly Landau-damped waves can propagate), even if $T_e=T_i$, provided $q_i^2 n_i \gtrsim (16-\gamma) q_e^2 n_e$ and $m_i/m_e \gg 16$.

Ion temperature gradient modes are also described by Eq. (13). The simplest version of the ITG dispersion relation neglects ω^* and β (assuming large ω_{Ti}^*),[10] in which case the resulting cubic equation for ω is $\omega'^3 = c_s^2 k_z^2 (\omega' - \ell \omega_{T_i}^*)$, a modified ion sound wave dispersion. The $\ell \omega_{T_i}^*$ term arises from the $\delta v_r \partial T_i/\partial r$ term in Eq. (5b), which describes temperature variations due to radial $E\times B$ convection of fluid elements in the presence of a temperature gradient. A solution of the cubic dispersion relation implies unstable waves exist if

$$\ell^2 \omega_{Ti}^{*2} > \frac{4}{27} c_s^2 k_z^2, \quad (15)$$

provided that Eq. (3) is also satisfied, so that ion and electron Landau damping is negligible. Inequality (15) can be rewritten as

$$k_z r < \frac{3\sqrt{3}}{2} \frac{\rho_i}{L_{Ti}} \frac{\bar{v}_i}{c_s} \sqrt{1 - \gamma \frac{\bar{v}_i^2}{c_s^2}}, \quad (16)$$

where $L_{Ti} \equiv T_i(\partial T_i/\partial r)^{-1}$ is the ion temperature gradient scale length and ρ_i is the ion cyclotron radius. At the maximum value of k_z for instability, $c_s k_{z_{\max}} = 3\sqrt{3}\ell \omega_{Ti}^*/2$, Eq. (13) implies $\omega' = 3\ell \omega_{Ti}^*/2$. Combining these values with the lower inequality in Eq. (3) implies $c_s/\bar{v}_i \gtrsim 4\sqrt{3}$, and applying this to Eq. (16) implies

$$k_z r \lesssim \frac{\sqrt{3(48-\gamma)}}{32} \ell \frac{\rho_i}{L_{Ti}}. \quad (17)$$

This can be satisfied in a sufficiently long plasma column. For example, in an H^+-positron plasma with $T_i = 10$ eV, $r = L_{T_i} = 1$ cm, $\gamma = 3$, and $k_z = \pi/L$ (the longest wavelength mode), instability occurs for $\ell L \gtrsim 270$ cm$(B/1$ Tesla). A more detailed analysis of frequency and stability of ITG modes in a nonneutral plasma will be considered in future work.

Drift waves are also described by Eq. (13). The elementary drift wave solution is found by assuming that $\omega_{Ti}=0$ and $k_z c_s/\omega' \ll 1$, implying

$$\omega' = -\ell \omega^* - \frac{i\beta\omega'}{1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2}. \quad (18)$$

The condition $4k_z \bar{v}_i \lesssim \omega'$ for weak Landau damping on the ions leads to a condition on k_z for drift waves similar to that for ITG modes,

$$k_z r \lesssim \frac{1}{4} \frac{T_e}{T_i} \frac{q_i^2 n_i}{q_e^2 n_e} \frac{\ell \rho_i}{L_{ni}} / (1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2), \quad (19)$$

where $L_{ni}^{-1} = n_i(\partial n_i/\partial r)^{-1}$ is the ion density gradient scale length. Like ITG modes, this condition can be met in a sufficiently long plasma column, or sufficiently large ℓ, and also becomes easier to meet when $n_e/n_i < 1$.

Drift waves in nonneutral plasmas can either be stable or unstable depending on the equilibrium density and electron temperature profiles. For example, in a collisionless drift wave the out-of-phase electron response is[9]

$$\beta = \sqrt{\frac{\pi}{2}} \frac{\omega - \ell(\omega_E + \omega_{De} - \omega_{Te}/2)}{|k_z \bar{v}_e|}, \quad (20)$$

assuming the "electrons" are light enough to neglect polarization drift and FLR effects, so that the drift-kinetic approximation is valid. Then according to Eq. (18) the local growth rate γ_{local} is given by

$$\gamma_{\text{local}} = \sqrt{\frac{\pi}{2}} \frac{\ell^2 \omega^*}{1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2} \frac{(\omega_i - \omega^* - \omega_E - \omega_{De} + \omega_{Te}/2)}{|k_z| \bar{v}_e}. \quad (21)$$

When the plasma is in a state of confined thermal equilibrium,[7] this local growth rate is negative, as required by stability of thermal equilibrium states. This can be seen by noting that the thermal equilibrium state is defined by $\omega_{T\alpha}=0$ (no temperature gradient) and $\omega_i + \omega_{Di} = \omega_E + \omega_{De} = \omega_F = \text{const.}$, where ω_F is the fluid rotation rate of the plasma including the diamagnetic drift. [Here we have employed a drift approximation for ω_F, neglecting some relatively small terms of order $O(\rho_i^2/L_{ni} r)$ associated with the centrifugal drift caused by the diamagnetic rotation.] Writing γ_{local} in terms of ω_F, using $\omega^* = (\omega_F - \omega_i) q_i^2 n_i T_e / [q_e^2 n_e T_i (1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2)]$ yields the negative-definite expression

$$\gamma_{\text{local}} = -\sqrt{\frac{\pi}{2}} \frac{q_i^2 n_i T_e}{q_e^2 n_e T_i} \frac{\ell^2 (\omega_i - \omega_F)^2}{|k_z| \bar{v}_e (1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2)^2} \times \left(1 + \frac{q_i^2 n_i T_e / q_e^2 n_e T_i}{1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2}\right), \quad (22)$$

consistent with the fact that drift waves must be stable in a thermal equilibrium nonneutral plasma.

However, when the plasma is not in thermal equilibrium, the drift waves can be unstable. In general, Eq. (21) can be expressed as

$$\gamma_{\text{local}} = \sqrt{\frac{\pi}{2}} \left(\frac{q_i}{q_e^2} \frac{c}{Br} \frac{T_e}{n_e}\right)^2 \frac{\ell^2}{|k_z| \bar{v}_e (1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2)^2} \frac{\partial n_i}{\partial r} \times \left[-\frac{q_e}{q_i} \frac{\partial n_e}{\partial r} - \frac{1}{1 + k^2\lambda_e^2 + k_\perp^2 \rho_s^2} \frac{\partial n_i}{\partial r} + \frac{q_e n_e}{2 q_i T_e} \frac{\partial T_e}{\partial r} - \frac{m_i q_e^2}{q_i^2} n_e r \frac{\omega_E^2}{T_e} \right]. \quad (23)$$

The four terms in the square bracket arise, respectively, from electron and ion diamagnetic drifts, the electron thermal diamagnetic drift, and the ion centrifugal drift. Only the second term is always stabilizing; the other three terms contribute to instability respectively when

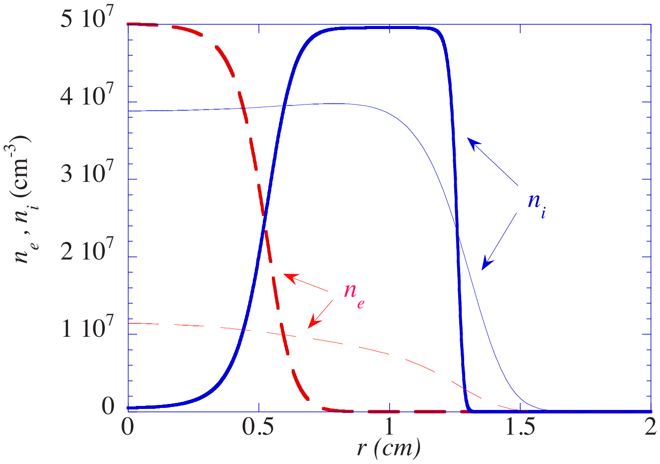

FIG. 1. (Color online) Electron (dashed) and ion (solid) densities in an $H^- - e$ plasma in thermal equilibrium for two temperatures, $T=300$ K (thick lines) or $T=1$ eV (thin lines), and assuming $B=0.5T$. When T is raised from 300 K to 1 eV the interface between species in the $T=300$ K profile becomes drift-wave-unstable.

$$\frac{\partial n_i}{\partial r}\frac{\partial n_e}{\partial r}<0, \quad \frac{\partial n_i}{\partial r}\frac{\partial T_e}{\partial r}>0, \quad \text{and} \quad \frac{\partial n_i}{\partial r}<0. \tag{24}$$

As is usual for drift waves, the most unstable modes will be those for which $k_\perp \sim \sqrt{\ell^2/r^2+k_r^2}$ is large and k_z is small (but nonzero). However, in nonneutral plasmas far from the Brillouin limit, $\lambda_e^2 \gg \rho_s^2$ or ρ_i^2 so Eq. (23) implies that the maximal growth rate will be for $k_\perp \lambda_e \sim 1$, not $k_\perp \rho_i \sim 1$. This implies that FLR effects and the polarization density term are not important for drift waves in such a nonneutral plasma, as opposed to neutral plasmas where typically $\lambda_e^2 \ll \rho_s^2, \rho_i^2$. Thus, the only important $O(1/B^2)$ term in Eq. (23) is the centrifugal term, the fourth term in the square bracket. It dominates over the other $1/B^2$ terms that were dropped, provided $\lambda_e^2/rL_n \ll 1$. Also, in a neutral plasma $q_e \partial n_e/\partial r + q_i \partial n_i/\partial r \simeq 0$, implying partial cancellation between the first two terms in the square bracket (for long wavelengths). This near cancellation does not generally occur in nonneutral plasmas.

In experiments it should be possible to manipulate the nonneutral plasma temperature and density profiles to turn the drift wave instability on and off at will. For example, if one starts with a plasma in thermal equilibrium, the drift wave is stable. If this equilibrium is at sufficiently low temperature so that

$$\frac{m_i \omega_F^2 r_p^2}{2} \gtrsim T, \tag{25}$$

where r_p is the plasma radius, then the ions will be centrifugally separated from the electrons, forming a hollow ring around the electron density.[7,8] At the interface between the species is a region where $\partial n_e/\partial r\, \partial n_i/\partial r < 0$, but this unstable term in the drift wave growth rate is balanced by the second and fourth terms in the square bracket of Eq. (23) to produce a stable mode. However, if the plasma is now heated, the stabilizing influence of the last term is reduced and the interface between the species can become drift-wave-unstable.

An example is shown in Fig. 1. Here an H^--electron (or H^+-positron) plasma at room temperature, $T=300$ K, is confined in thermal equilibrium with a central electron density of 5×10^7 cm^{-3} and a radius of about 1.3 cm. These density profiles were obtained by solving the coupled Poisson–Boltzmann equations for two charge species in cylindrical geometry, Eqs. (6)–(8) of Ref. 8. At a magnetic field of 0.5 Tesla, the ions are centrifugally separated due to the roughly 10^6 rad/s rotation frequency of the plasma.

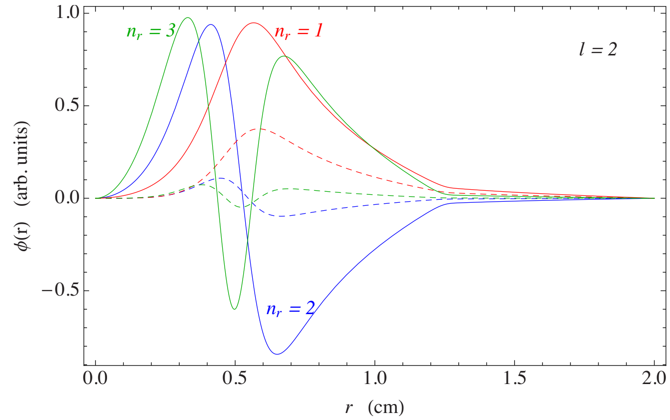

FIG. 2. (Color online) Three unstable drift wave eigenmodes for the plasma densities shown in Fig. 1 (with thick lines), taking $T=1$ eV and $B=0.5T$, for $\ell=2$ and $n_r=1$, 2, and 3. Modes are found by solving Eq. (26). Dashed lines are the imaginary part and solid lines are the real part.

If the temperature were now raised to 1 eV, Eq. (25) would no longer be satisfied and the electron and ion densities would relax to a new thermal equilibrium state where the ions and electrons are more uniformly mixed (the second set of density profiles in Fig. 1).[7] The $T=1$ eV profiles were found by determining the thermal equilibrium state with the same particle numbers and total canonical angular momentum as the $T=300$ K equilibrium (assuming that the temperature was raised without adding particles or torquing on the plasma).

The relaxation of the density profiles toward a $T=1$ eV thermal equilibrium state can occur either via col-

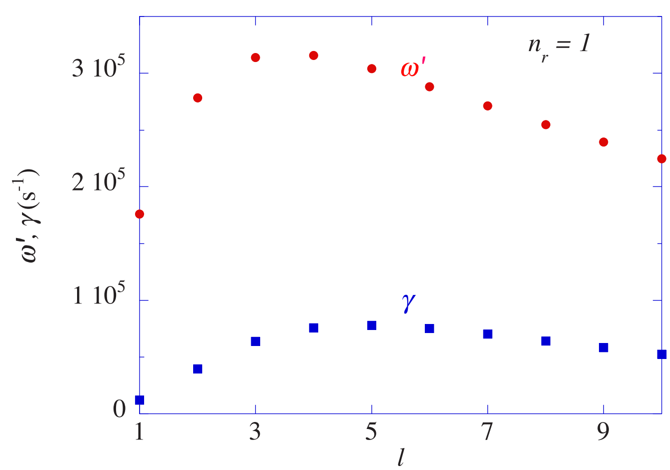

FIG. 3. (Color online) Real and imaginary parts of the drift wave frequency for the same plasma as in Fig. 2 vs azimuthal mode number ℓ, for radial mode number $n_r=1$. Here, $\omega'=\omega-\ell\omega_i(r)$ is plotted at $r=0.5$ cm, so as to subtract out the large rotational Doppler shift in the mode frequency. At $r=0.5$ cm, $\omega_i(r)=9.24\times10^5$ s^{-1}.

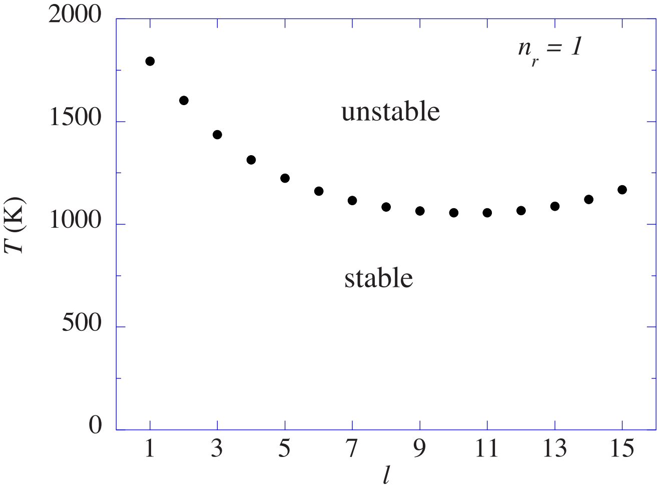

FIG. 4. (Color online) Stability diagram for drift modes for the same plasma as in Fig. 2 as temperature is varied, for radial mode number $n_r=1$.

lisional processes or instability. Here, we note that the centrifugally separated density profiles in Fig. 1, which were stable thermal equilibrium profiles at $T=300$ K, become drift-wave-unstable when T is raised to 1 eV. This can be seen by computing drift wave eigenmodes numerically using the shooting method to solve

$$\nabla_\perp^2 \delta\phi = \frac{\delta\phi}{\lambda_e^2}(1+i\beta) + \frac{4\pi q_i \ell c}{Br}\frac{\partial n_i/\partial r}{\omega - \ell\omega_i(r)}\delta\phi, \qquad (26)$$

with β given by Eq. (20). This simplified drift wave equation follows from Eqs. (4) and (10), dropping the polarization drift terms and assuming $k_z \bar{v}_i/\omega' \ll 1$ in order to simplify the ion response. Some representative eigenfunctions are shown in Fig. 2. The modes are radially localized to the interfacial region between species by the effect of electron Debye-shielding. This shielding effect vanishes as $n_e(r)\to 0$, so the mode potential is nonzero outside the plasma and could be picked up using probes or wall patches. Frequencies and growth rates are displayed in Fig. 3 for several modes, assuming $k_z=\pi/100$ cm. All these modes satisfy Eq. (3).

The local approximation, Eq. (18), provides qualitatively similar results to the numerical solution of Eq. (26). For instance, Eq. (18) predicts a maximum in ω' when $\ell \sim r/\lambda_e$, as observed in Fig. 3 if one takes $r/\lambda_{\mathrm{de}} \sim 3-5$ to be the rough location of the eigenmode. Also, as expected from the expression for the local growth rate, modes with larger ℓ and lower radial mode number n_r grow more rapidly, up to $\ell \sim 5$.

These unstable modes would be expected to rapidly saturate and produce transport that leads to mixing of the ions and electrons, driving the plasma toward the $T=1$ eV thermal equilibrium state shown in Fig. 1. In the absence of such turbulence, this mixing takes a rather long time: for the parameters of the example, the collisional electron-ion diffusion coefficient is roughly $D \simeq 10^{-3}$ cm^2/s, so it would take many seconds for the density profiles to collisionally relax.[11–13] One would expect the drift wave turbulence to greatly enhance this relaxation rate. Cooling the plasma back to 300 K would then return the ions to the plasma periphery, through either collisional relaxation or another instability, and the experiment could be performed again. This would allow repeatable and reproducible measurements of drift instability and transport in a confined plasma column.

These results suggest a second experiment, where for the previous thermal equilibrium density profiles at $T=300$ K and $B=0.5T$, the temperature is increased starting from 300 K until the drift modes begin to go unstable. In Fig. 4 we display the predicted stability diagram for modes with $n_r=1$, which are the most unstable modes, according to Eq. (26). The least stable mode has $\ell=9$ and goes unstable at a temperature of $T=1055$ K.

In conclusion, we have briefly described how nonneutral plasmas consisting of two or more species can exhibit ion sound waves, drift waves, and ion temperature gradient waves, provided that certain conditions are met. We observed that weakly Landau-damped ion sound waves can propagate in a collisionless nonneutral plasma even if $T_e=T_i$, provided that $q_i^2 n_i/q_e^2 n_e \gtrsim 16-\gamma$ and $m_i/m_e \gg 16$. We also briefly examined conditions under which drift and ITG waves can exhibit instability, demonstrating how such conditions could be met in current nonneutral plasma experiments. This could allow study of ITG and drift wave instabilities and the resulting anomalous transport under carefully controlled conditions.

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