C. PROJECT DESCRIPTION

1. NSF/DOE Partnership Support

Intellectual Merit. This long-term collaboration between Driscoll, Dubin, and O’Neil has developed theory and experiment on a wide range of fundamental processes in plasmas. The conceptual simplicity, long confinement times and excellent control of cylindrical non-neutral plasmas has enabled incisive quantitative comparison of theory and experiment. The high quality of the research has been recognized through many invited and plenary session lectures and through major APS awards.

An important aspect of our experiments is the ability to vary plasma temperature over several decades, accessing different physical effects in different regimes. An example is plasma wave damping, measured in both electron and ion plasmas, over 3½ decades in temperature (Fig.1). At high temperature the waves are Landau damped; at lower temperatures bounce-harmonic effects dominate, then interspecies collisional drag, then strong magnetization and strong-coupling effects. Much of our proposal involves investigations of these different effects.

The interplay between theory and experiment is one of the main strengths of our research: experiments suggest new avenues for theory (e.g. flux-driven damping of drift modes) and theory focuses experiments on new effects (e.g. long-range collisional enhancement of plasma wave damping). Overall, the research program is fairly broad, as befits a program with three active PI’s. The proposal describes research on linear and nonlinear waves; on long-range collisions and strong coupling; on separatrix dissipation and neoclassical transport; and on 2D vortex dynamics. The prior research (2013-2016) resulted in 36 publications, listed in the References, and described herein (Table 1) as context for the proposed research.

Broader Impacts The research has strong interdisciplinary connections, contributing to and borrowing from the wider world of plasma physics, atomic physics, fluid dynamics, astrophysics, and statistical physics. Graduate and undergraduate education has been a major component of the NSF grant. Four graduate students are typically supported, 2 theory and 2 experiment. During the prior period, 3 Ph.D.s were completed. The group mentors undergraduates in short-term research projects, and outreach to high school students occurs through lab tours and special lectures.

NSF Phy-1414570 $300,000/yr, Jul-2014 - Jul-2017 Fundamental Processes in Plasmas
DOE DE-SC0002451 $400,000/yr, Jul-2014 - Jul-2017 Fundamental Processes in Plasmas

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2a. CamV: Camera-diagnosed electron plasmas

The CamV apparatus utilizes a CCD camera to provide quantitative images of electron plasma density \( n(r, \theta, t) \), and temperature \( T(r, t) \), for both 2-dimensional (2D) \( \mathbf{E} \times \mathbf{B} \) flow phenomena, and for 3D transport and damping effects associated with controlled magnetic and electrostatic separatrices.

The unneutralized electron plasma is trapped in a series of 15 cylindrical electrodes in an ultra-high vacuum environment (Fig. 2). The electrons are contained axially by negative voltages on two end cylinders, and confined radially by a uniform axial magnetic field (\( B < 2 \) Tesla), resulting in a confinement time of about 100 sec. The trapped electron column typically has density \( n \approx 10^7 \) cm\(^{-3}\), radius \( R_p \approx 1.5 \) cm, and axial length \( L_p \approx 50 \) cm, with temperatures in the range \( 0.03 \text{ eV} \leq T \leq 3 \text{ eV} \). Individual electrons bounce rapidly axially with low collisionality \( v_{\perp} \approx 10^{-10} \text{ sec}^{-1} \), and \( \mathbf{E} \times \mathbf{B} \) drift across the magnetic field.

At any desired time, the \( z \)-integrated electron density \( n(r, \theta, t) \) is measured (destructively) by dumping the electrons axially onto a phosphor screen, which is imaged by a low-noise CCD camera. The 3D density \( n(r, \theta, z, t) \) and self-consistent potential \( \phi(r, \theta, z, t) \) can be constructed by assuming a Boltzmann equilibrium along each field line. The temperature \( T(r, t) \) can be obtained from partial dumps over controlled electrostatic barriers. The shot-to-shot variations in the initial profiles are small, so the time evolution can be obtained from a sequence of shots with differing hold times. CamV incorporates about 7 racks of control and diagnostics electronics, as well as an extensive set of custom software programs, enabling fully quantitative measurements of subtle wave and transport effects.

The \((r, \theta)\) flow of “\( z \)-bounce-averaged” electrons is described by the 2D drift-Poisson equations, which are isomorphic to the Euler equations for an incompressible inviscid 2D fluid, with true free-slip radial and axial boundary conditions [167-171]. Only for fine spatial scales or long times (i.e., seconds), do collisional “viscous” [130-134] or diffusive effects [124-127] become significant. Major results include (the asterisks denote the first measurements of a specific effect):

*Formation of 2D turbulence to vortex crystals[180-183] — vortex dynamics in background vorticity[182] — inviscid symmetrization of vortices [174]  
* fluid echoes from spatial Landau damping [184]

Contrasting with the inviscid fluid perspective is the kinetic perspective, including subtle dissipative effects from separatrices induced by localized particle trapping. The resulting collisional and “chaotic” neoclassical transport (NCT) has recently been a primary research focus:

*Chaotic NCT in parametric decay of drift waves[21]  
*Separatrix-induced damping of diocotron waves[22,156] and \( \theta \)-asymmetric BGK plasma waves [157]  
— Superbanana particle transport in the low-collisionality regime [20, 22, 156, 158-160,162]

**EV:** The older EV apparatus contains electron plasmas similar to those of CamV, with a lower magnetic field of \( B < 0.067 \) Tesla. The interior electrodes enable similar manipulation and diagnostics, but the dump diagnostic is limited to a radial scan. EV does, however, have a unique secondary magnet which allows measurement of the perpendicular temperature. The EV apparatus is typically used for student training, and to extend and verify results from another apparatus.
2b. IV: Laser-diagnosed ion plasmas

The IV apparatus contains unneutralized ions rather than electrons, so that sophisticated Laser-Induced Fluorescence techniques can be used for diagnostics and control. The Mg+ ion plasmas have density \( n = 10^6 - 10^8 \text{ cm}^{-3} \), radii \( R_p = 0.5 - 1 \text{ cm} \), and lengths \( L_p = 1 - 30 \text{ cm} \), with thermal energies ranging over \( T = 10^{-5} - 10^1 \text{ eV} \), in a magnetic field \( B \leq 3 \text{ Tesla} \).

The ions are confined in steady-state for days or even weeks using the “rotating wall” technique [33, 49-59], allowing transport measurements with unprecedented control and precision. This technique has now been adopted by 27 institutions world-wide [63-80], including the anti-matter recombination experiments at CERN [60-62]. By varying temperature over 5 decades and density over 2 decades, the IV experiments provide strong tests of theory in many plasma regimes.

LIF diagnostic techniques allow time- and space-resolved measurement of density, temperature and rotation velocity; moreover, “test” particles can be “spin-tagged” and followed in time. A second beam is used to cool or heat the plasma or to manipulate the radial profile. Figure 3 shows the laser table: two tuneable CW dye lasers that are pumped by two solid state 10W 532 nm lasers. The dye lasers at 560 nm are frequency-doubled with two custom frequency-doubling cavities, producing two 10mW beams at \( \sim 280 \text{ nm} \), which can be directed either along or across the plasma column.

The frequency stability of the two lasers has recently been improved, to allow diagnostics and control to \( T \leq 10^{-6} \text{ eV} \). A sophisticated wavemeter (High Finesse WS-2) and feedback laser control reduces the laser frequency variability to \( \Delta f / f \leq 4 \times 10^{-9} \), corresponding to \( ^{24}\text{Mg}^+ \) thermal velocity at \( 4 \times 10^{-7} \text{ eV} \). This extends our operating range from plasma (\( \Gamma < 1 \)) to liquid (\( \Gamma = 1 \)) to crystal (\( \Gamma \geq 200 \)), extending our transport measurements into the correlated regime (Sec. 4c,d).

The IV apparatus was developed with ONR equipment funding of approximately $1.4M from 1990-1998. Continuing operation in our education-oriented program is primarily funded by the NSF/DOE Partnership in Basic Plasma Science. The frequency stability upgrade was funded by the DOE High Energy Density Laboratory Plasma program.

Since inception, the IV apparatus has enabled quantitative comparison to theory on a broad range of basic plasma effects. The asterisks denote the first (or only) measurements of a specific effect.

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* Cyclotron modes in multi-species plasmas [1,7,8,9,90]  — Thermally-excited Langmuir modes [185]
* Rotating Wall \( \infty \)-time confinement [49,50]  * Salpeter Enhancement of Close Collisions[16,187,188]
* Diffusion from Long-Range Collisions [124,125]  — Linear Landau damping & Trapping osc’s [186]
* Shear Reduction of collisional transport [126]
* Verification of enhanced collisional drag from long-range collisions[19]
* Bounce-harmonic Landau damping of TG waves[14]
3. Plasma and Cyclotron Waves

3a. Cyclotron Waves, Bernstein Modes

We have recently developed theory [5] and experiments [1,7,8,9] characterizing cyclotron mode frequency shifts in multi-species ion plasmas, due to external E-fields and collective plasma effects. Measurement of the frequency shifts allows a non-destructive determination of the mass and concentration of each plasma species. Quantitative concentration measurements enabled our verification of enhanced collisional drag damping of magnetized plasma waves (see Sec. 4b). This comprised a portion of the Ph.D. thesis of M. Affolter.

The applied physics, chemistry, and biology communities use cyclotron resonance spectroscopy devices to determine atomic and molecular masses and compositions, often with ad hoc corrections for electric field and collective effects [81-83]. For cyclotron modes varying in $\theta$ as $\exp(\text{im} \theta)$, the actual cyclotron frequency $f_m^{(s)}$ on each species $s$ is shifted away from the “bare” cyclotron frequency $2\pi f_c^{(s)} = q_sB/M_sc$ by trap electric fields, by space charge [84,85], and by (possibly amplitude-dependent) plasma effects; our work rigorously evaluates and measures these effects.

For radially uniform multispecies plasmas, the theory predicts cyclotron frequency shifts proportional to the ExB rotation frequency $f_E$ and also the species fraction $\delta_s$, as

$$f_m^{(s)} - f_c^{(s)} = \left[(m-2) + \delta_s \cdot (1 - R_m)\right] \cdot f_E. \tag{1}$$

Here, the wall image charge correction is $R_m = (R_p/R_o)^{2m}$ for $m \geq 0$, and $R_m = 0$ for $m = 0$. Recent experiments [1,7,8,9] have quantitatively verified Eq. (1) over the broad temperature range where the ion plasmas are radially uniform, for $m = 0, 1, 2$. Figure 4a shows mode resonances for $^{24}\text{Mg}^+$ points, compared to Eq. (1). The space–charge and collective effects were (purposefully) large, but well characterized by the LIF measurement of $f_E$. Absent laser diagnostics, one can determine $f_E$ and $\delta_s$ from the cyclotron frequencies for multiple $m$ modes. This method allows higher accuracy determination of ion masses and concentrations than single point methods. For example, given just the 4 circled $f_m^{(s)}$ data points in Fig. 4a, the two $f_E$ values are determined to within 1%, and the species concentration $\delta_s$ to within 10%.

At lower temperatures, plasma rotation causes ion species to radially-separate due to centrifugal effects [86,62,33], producing frequency shifts that are predicted with a more detailed theory accounting for radial non-uniformity of the plasma [87, 5] (Fig. 4b). At very low temperatures, each species tends towards a radially-localized annulus, each with the same rotation rate. Each species then has radially-localized cyclotron modes as if $\delta_s = 1$, and the observed frequency shifts are reduced toward zero [7,8].

We have also developed the method of Thermal Cyclotron Spectroscopy (TCS) [7], which provides faster and more accurate species concentration measurements, down to the 0.1% range. A series of RF bursts, scanned over frequency $f$, are applied to wall sectors to create an $m=1$ drive. Resonant wave absorption then heats the plasma, changing the $^{24}\text{Mg}^+$ velocity distribution, which is detected through LIF diagnostics (Fig. 5). For short bursts compared to the collision time, the energy in the resonant cyclotron motion scales as $M_s (q_sE_0/M_st)^2 \delta_s$ where $E_0$ is the $m=1$ burst amplitude. Thus,
the ratio between two species concentrations can be determined by the ratio between the plasma temperature changes induced from equal-time bursts at the two species cyclotron frequencies: \( \Delta T_{s_1} / \Delta T_{s_2} = (\delta_{s_1} / \delta_{s_2})(M_{s_2} q_{s_1}^2 / M_{s_1} q_{s_2}^2) \). Concentrations evaluated using TCS agree with those measured using the previously-described frequency fits to the surface cyclotron resonance.

**Bernstein modes.** Our group has also recently developed a kinetic theory and an accompanying numerical analysis [5] for linear electrostatic waves near the cyclotron frequency \( f_c(s) \) of any given species \( s \). Building on prior analyses [87-90], the theory and numerics keep non-uniform radial density, finite cyclotron radius corrections and non-Maxwellian distribution corrections. The results describe surface cyclotron waves varying as \( \exp(\imath m \theta) \), and also predict coupling of these waves to radially and axially propagating finite-temperature Bernstein waves \([91,92]\) that modify the plasma response to applied rf wall signals. These Bernstein waves produce a new set of normal modes in the plasma column as they propagate radially between center and edge.

These modes must be accounted for in order to properly interpret the plasma response to an applied wall signal at frequency \( f \) as measured, for example, by the scaled admittance function \( Y = R_q \, \text{Im}(\delta \phi / \delta R_q) / \delta \phi (R_q) \) [5,87] where \( \delta \phi (r) e^{\imath (m \theta - \omega t)} \) is the perturbed potential and \( \omega = 2\pi f \). Without this improved understanding a set of resonance peaks produced by Bernstein modes in a single species plasma could be incorrectly interpreted as separate cyclotron resonances due to different species with closely-spaced charge/mass ratios (see Fig. 6). Bernstein normal modes were observed previously in an electron plasma [93] but could not be carefully compared to theory because the plasma density and temperature profiles were not measured. We propose to test our new theory for these Bernstein modes against experiments on well-characterized IV plasmas over a range of temperatures and magnetic fields.

We also propose to measure the damping rate of these cyclotron/Bernstein modes. Unlike some past work on damping of such modes[94], Landau damping on the parallel velocity due to a “magnetic beach” [95] is not operative because the magnetic field is uniform and the modes are (nominally) \( z \)-independent. Instead, there is (in theory) spatial Landau damping somewhat similar to that for diocotron waves ([96],Sec. 6a), and collisional damping. Unlike TG modes (Sec. 4b, Fig. 1), collisional damping of cyclotron/Bernstein modes should be strongly reduced in regimes where the plasma becomes strongly magnetized (i.e. when the cyclotron radius satisfies \( r_c < e^2 / T \) ; see Sec. 4d), since cyclotron action is conserved in this regime[97]. Using a time-resolved version of TCS, the collisional drag damping of the \( m=1 \) cyclotron mode should be observable over a wide range of plasma parameters; some initial measurements are shown in Fig. 4b (as the error bars). Velocity-space instabilities could also occur during TCS [98] and this is an open question that we propose to address.

**3b. Enhanced Bounce-Harmonic Landau Damping from Potential Variations**

We have studied the enhanced “bounce-harmonic” Landau-damping of long-wavelength Trivelpiece-Gould (TG) oscillations, caused by potential variations along the plasma column [2,6,11,14]. This damping mechanism dominates over other processes at temperatures somewhat below the regime of normal Landau damping (Fig. 1).

Landau damping arises from resonant interactions between a traveling wave with axial phase velocity \( v_e = 2\pi f / k_z \) and nominally free-streaming charges with velocities near \( v_\parallel \). For standing TG waves on a finite-length plasma column, hypothesizing “specular reflection” at the ends preserves the Landau resonance at \( f = f_h(E) \) for particles with energy \( E = mv_\parallel^2 / 2 \) bouncing end-to-end at frequency \( f_h = v_\parallel / 2L \). The Landau damping predicted from this resonance is the red curve in Fig. 1.

However, introducing realistic \( z \)-dependent end potentials, or other potential variations along the column, causes particles to speed up and slow down rather than simply reflect from the ends. This introduces...
extra Fourier time-harmonics in the particle trajectory relative to the wave phase, causing “bounce-harmonic” Landau-damping from particles satisfying \( f_0(E) = f / n \) for integer \( n \) [99,100].

Our recent theory work analyzed these bounce-harmonic effects in detail [2,6,11], motivated by the “bigger-than-Landau” damping measurements displayed in Fig.1 for 0.003 < \((\nabla / v_φ)^2\) < 0.03. Although bounce-harmonic Landau damping has been inferred from previous observations in mirror machines [101,102], our recent experiments [14] have provided a precise, direct test of the bounce-harmonic Landau damping theory. The theory and experiments comprised a portion of the Ph. D. theses of A. Ashourvan and M. Affolter respectively.

In our experiments, a controlled potential variation is created by applying a “squeeze” voltage \( V_{sq} \) to a cylindrical electrode near the center of the plasma column. Fig. 7 shows the enhancement of the damping (over the background rate \( γ_0 \)) for the TG mode with \( k_c = \pi / L \), displaying data from both electron and ion plasmas. The measurements are in good agreement with theory which predicts that the \( n=3 \) bounce harmonic dominates the damping for these measurements. As shown in the inset, the damping is independent of the sign of the squeeze potential, consistent with the predicted \( V_{sq}^2 \) dependence. This fact demonstrates that particle trapping and separatrix dissipation effects (Sec. 5) are not dominant here, since these trapping effects depend on the sign of the squeeze.

The perturbed velocity distribution function \( \delta F \) is also measured using phase-coherent LIF detection (Fig. 8). Our data analysis divides \( \delta F(v) \) into symmetric and antisymmetric parts in \( v \); as expected, the entire effect is antisymmetric, and shows the resonance due to the \( n = \frac{3}{3} \) bounce harmonic at particle velocity \( v = v_φ / 3 \).

Fortunately, this controlled-squeeze damping is much easier to calculate theoretically than the bounce-harmonic Landau damping \( γ_0 \) present when no squeeze is applied, caused by particles reflecting off of the plasma end potentials (as in Fig. 1). This is because the applied squeeze field makes only a small and calculable correction to the plasma orbits; and the wave field in the region of the squeeze is well—understood using perturbation theory [6,11]. In contrast, no perturbation theories apply in evaluating the wave field near the plasma ends, which is which is required to evaluate \( γ_0 \). This remains a surprisingly difficult open problem that we will continue to work on.

On the other hand, for nonlinear TG waves (BGK states) varying as \( \exp(i m \theta) \), preliminary theory suggests that there are still large gradients in the velocity distribution function at the squeeze separatrix, caused by bounce-averaged radial ExB drifts in the wave potential. Thus, for these nonlinear waves there may still be an active separatrix dissipation mechanism. Previous experiments [103] have observed nonlinear wave damping consistent with such separatrix dissipation when \( m > 0 \) (linear waves are typically Landau-damped in the experimental regime). We propose to study this further to distinguish when Landau resonances are more or less important than separatrix dissipation in damping these waves.

3c. Parametric Decay of TG Modes

Parametric decay is a fundamental nonlinear process important in a great many physical systems [104-108]. Our experiments are currently pursuing direct comparisons between theory, simulations, and experiment on the parametric decay of standing TG modes to longer wavelength, e.g. \( k_c = 2(\pi / L) \) decaying to \( k_c = 1(\pi / L) \).
Of particular interest here is that for a finite radius plasma column, TG waves have acoustic dispersion for small $k_z$, enabling the simplest 3-wave decay ($f_i = f_1 + f_2$, $k_z = k_1 + k_1$). However, this also implies that standard three-wave theory[109-111] for the decay may be insufficient, since resonances between several standing waves (or wave harmonics) are possible [10]. The wide temperature variation available in our experiments ($1 < v_p / v < 100$) allows study of such decay in both the fluid and the kinetic regimes.

Wave coupling effects are mitigated by detuning between the discrete standing modes, quantified by $\Delta = f_2 - 2f_1$. Our experimentally accessible detuning range is approximately $0.02 < \Delta / f_2 < 0.20$, using both CamV and IV. Standard three-wave fluid theory predicts exponential $2 \rightarrow 1$ decay at rate $\Gamma_s$ for small detuning, with an abrupt transition to oscillatory coupling for larger detuning. Experimentally, the transition region displays a broad range of effects: from no decay, to slow decay, to intermittent decay[12]. Some of these effects are predicted by a more detailed cold fluid theory that goes beyond the 3-wave approximation, keeping multiple near-resonant standing waves [10]. However, the observed parametric decay rates for TG waves are larger than the current multi-mode cold-fluid theory predictions, while simulations and standard three-wave theory are in better agreement with the preliminary experiments (Fig. 9). This may be because the current multi-mode cold fluid theory neglects time-dependent thermal frequency corrections due to plasma heating in the presence of large standing waves, which changes the detuning, an effect captured in the kinetic simulations. This experimental and theoretical work comprised a portion of the Ph.D. theses of M. Affolter and A. Ashourvan respectively.

We propose to make a broad survey of nonlinear coupling and decay rates for TG waves, for direct comparison to improved fluid and kinetic theory and simulations.

3d. Waves in Electron–$H^-$ Nonneutral Plasmas

Recently, accelerator-generated antiprotons have been trapped and cooled together with electrons in nonneutral plasma traps, in order to synthesize cold antihydrogen atoms[62,112]. These disparate-mass nonneutral plasmas exhibit several novel plasma wave effects, but the price tag and lifetime of the antiprotons have so-far limited any extensive studies.

We propose to create and characterize electron/$H^-$ plasmas in the CamV apparatus. We intend to study several low-frequency plasma waves that have been predicted to exist in such plasmas[3,113]: ion sound waves, drift waves, and ion temperature gradient waves. These waves have never been observed in nonneutral plasma experiments, because the waves require a large ratio between species masses: a light species with nearly-adiabatic response to perturbations is required, along with a heavy species with a nearly cold-fluid response. Theory predicts that these low frequency waves have dispersion relations similar, but not identical, to those in quasineutral plasmas. For example, drift waves are not necessarily unstable in a nonneutral plasma, but can be driven unstable by centrifugal effects associated with nonthermal equilibrium density profiles[113].

These predicted effects become significant with a $H^-$ fraction of 10-20%, which we believe can be obtained in CamV. In preliminary experiments, we have accumulated $H^-$ in a room-temperature cold electron plasma using two-stage in-volume production. However, the accumulated fraction of $H^-$ so far is limited to a few percent due to the low $H^-$ creation rate and strong centrifugal mass separation [86] followed by transport of $H^-$ to the wall.

To increase the creation rate of $H^-$ we propose to insert a second source in the CamV machine: a negatively biased cesiated electrode, which converts a fraction of incident $H^+$ into ejected $H^-$ [114]. This source should produce a nearly constant (10-15%) $H^-$ ion yield per impinging positive ion, above a 5 eV energy threshold [115]. Characterization of the resulting plasma will then determine which wave experiments are practical.
4. Long-Range Collisions, Energy Equipartition

Our extensive program of theory and experiment has established that classical transport theory [116-119] grossly underestimates collisional plasma transport when the relevant cyclotron radius is small compared to the Debye length (i.e. $r_c << \lambda_D$ ). The increased transport is from long-range collisions, in which particles interact when separated by impact parameters $\rho$ in the range $r_c < \rho < \lambda_D$.

In short-range “classical” collisions, the particle velocity vector scatters, shifting the particle guiding center by $\Delta r = r_c$; such scattering occurs only for small impact parameter $\rho \leq r_c$. Experiments with neutral plasmas have investigated this classical transport in various regimes [120-123]. Long-range collisions have impact parameter $\rho > r_c$; for these collisions, the interaction electric field causes the particles to $E \times B$ drift across the magnetic field, and also to exchange parallel energy and momentum. The transport resulting from these long-range collisions can be orders of magnitude larger than classical transport; for example, long-range cross-magnetic-field heat transport is independent of magnetic field. Note that this enhanced transport is due to collisions and thermal fluctuations, not to turbulence. Although the experiments are performed on non-neutral plasmas, the implications of the theory extend to plasmas in general.

An overview of the predicted collisional cross-magnetic-field transport coefficients for diffusion [124-127], heat [128,129], and viscosity [130-135] is given in Table 1, assuming $r_c < \lambda_D$ . The 4th column describes newly-predicted enhanced collisional slowing parallel to $B$, discussed below. Entries colored green represent theory and experiment in good agreement; for entries in red, experiments have either not yet been completed, or have uncovered unresolved discrepancies with the theory.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Test Particle Diffusion $D$</th>
<th>Thermal Diffusivity $2k/3n$</th>
<th>Shear Viscosity $\eta/\eta_{\nu}$</th>
<th>Parallel Slowing $v_{\nu}$ $b &lt; r_c &lt; d$ assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-Range $\rho &lt; r_c$</td>
<td>$\frac{4}{3} \sqrt{\pi} v_0 r_c^2 \ln \left( \frac{r_c}{\rho} \right)$</td>
<td>$\frac{16}{9} \sqrt{\pi} v_0 r_c^2 \ln \left( \frac{r_c}{\rho} \right)$</td>
<td>$\frac{2}{5} \sqrt{\pi} v_0 r_c^2 \ln \left( \frac{r_c}{\rho} \right)$</td>
<td>$\frac{4}{3} \sqrt{\pi} \frac{2 \pi \mu}{m_s} v_0 \ln \left( \frac{r_c}{\rho} \right)$</td>
</tr>
<tr>
<td>Long-Range $r_c &lt; \rho &lt; \lambda_D$</td>
<td>$2 \sqrt{\pi} v_0 r_c^2 \ln \left( \frac{\lambda_D}{r_c} \right)$</td>
<td>$0.82 v_0 \lambda_D^2 + \text{waves}$</td>
<td>$0.59 v_0 \lambda_D^2 + \ln \left( \frac{\omega_c}{\gamma} \right)$</td>
<td>$\frac{2 \pi \mu}{m_s} v_0 \ln \left( \frac{d}{r_c} \right)$ + $2 \ln \left( \frac{\lambda_D}{d} \right)$</td>
</tr>
<tr>
<td>3D: $k_z \neq 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D: $k_z = 0$</td>
<td>$4 \pi v_0 r_c^2 \frac{f_0}{</td>
<td>S</td>
<td>} \ln \left( \frac{r}{\rho_m} \right)$</td>
<td>NA</td>
</tr>
</tbody>
</table>

$v_0 \equiv n e_b^2$; $b \equiv e^2 / T$; $\gamma = \text{Max}(S, \nu_c)$; $S = |te_E|$; $\rho_m \sim [4D / S]^{1/2}$; $f_0 = \gamma / 2 \mu / \mu$; $d = \left( e^6 / \mu^3 D_z^2 \right)^{1/5}$

4a. New Theory of Collisional Drag Parallel to $B$

The parallel slowing rate $v_{\nu}$ due to collisional drag between species $s$ and $s'$ is important to a number of processes, including the growth rate of non-ideal plasma instabilities, magnetic reconnection, and runaway electrons[136-138]; however, a precise theory of the rate had not been previously formulated for plasmas in the regime $r_c < \lambda_D$ . Our new theory[18] and experiments[19] show that the collisional slowing rate $v_{\nu}$ can be considerably enhanced by long-range collisions.

Two classical theories of long-range collisions, Boltzmann and Fokker-Planck, give different answers for the slowing rate from long-range collisions between guiding centers moving in one dimension (1D), parallel to $B$ [139]. In a Boltzmann collision, a colliding pair is assumed to be isolated; the total energy and parallel momentum are conserved. In a Fokker-Planck (FP) collision, many interactions are happen-
ing simultaneously, but each interaction is assumed to be weak.

For long-range Boltzmann collisions, two guiding centers either reflect from one another as they move along the magnetic field, or they pass by with their final velocities unchanged from initial velocities. Thus, only reflecting particles contribute to slowing down in the Boltzmann picture. For example, in electron-ion collisions Boltzmann analysis predicts no reflections (the Coulomb force is attractive), and hence such collisions have no contribution to the slowing down rate. In contrast, FP analysis predicts a finite rate for these e-i collisions.

Our new theory[18] predicts that the discrepancy between the Boltzmann and Fokker Planck models is resolved by consideration of a novel but fundamental length scale

\[ d = \left( \frac{c^2}{\mu} \right)^{1/3} D_1 \]

Here \( D_1 \) is the parallel velocity diffusion coefficient, related to \( v_w \) by the Einstein relation \( v_w = D_1 / T \); and \( \mu = m_m / (m_s + m_i) \). Collisions with impact parameter \( \rho < d \) that result in reflections are on average sufficiently rapid to be treated as isolated Boltzmann collisions. But if \( \rho > d \), reflections are slow enough so that velocity diffusion due to interactions with surrounding particles disrupts the reflection, requiring a FP theory analysis.

Thus, the theory predicts a slowing rate with two Coulomb logarithms as shown in Table 1. (Table 1 assumes \( b < r_c < d \) for simplicity; a more general expression may be found in [18].) The first logarithm arises from Boltzmann collisions with impact parameter \( \rho \) in the range \( r_c < \rho < d \), and the second from FP collisions with impact parameters in the range \( d < \rho < \lambda d \). Interestingly, the coefficient \( h \) in the Boltzmann term depends on the sign of the Coulomb interaction; for attractive interactions it is zero, but for repulsive interactions it is 5.899.

This computed value for \( h \) is enhanced over naïve 1D Boltzmann theory (by a factor of 47%) by a novel “collisional caging” effect: a repulsive Boltzmann collision between a particle pair does not occur only once; instead, it occurs as part of a correlated sequence. After one reflection, parallel velocity diffusion due to the interactions of the pair with surrounding charges may cause the relative velocity of the pair to reverse, so that the pair collides again. If the pair has sufficiently low relative velocity it can reflect several times consecutively. Each consecutive reflection produces the same sign of relative acceleration, increasing the correlation time, and thus increasing the collision rate. Eventually, velocity diffusion causes the relative velocity to become sufficiently large so that the particles pass by rather than reflect, after which the acceleration due to their interactions averages to zero, and their mutual correlation is lost.

4b. Drag Damping of Plasma Waves

Our recent experiments[19] on the IV apparatus have verified this enhanced drag due to long-range collisions. The experiments observe the damping rate of TG waves in a multi-species ion plasma. Over a range of about two decades in temperature (Fig. 1, Fig. 10) the damping is dominated by collisional drag between the plasma species. The heavy species reacts to the wave electric field less than a light species, and this difference in parallel velocities induces a frictional drag between species with damps the wave. For plasmas that are both radially uniform and have weak collisionality (\( v_w \ll \omega_{pi} \)), the collisional drag damping rate \( \gamma \) can be solved analytically as [140]

\[ \gamma = \frac{1}{4\omega_p^2} \sum_j \sum_{j'} \frac{(M_j - M_{j'})^2}{M_j^2} \omega_{p,j'}^2 v_{w,j'} \]

where \( \omega_{p,j'} = 4\pi e^2 n_{j'} / M_j \) is the species plasma frequency squared, and \( \omega_p^2 = \sum_s \omega_{p,s}^2 \) is the total plasma frequency squared. More generally, the damping rate is determined by numerically-solving the coupled linearized fluid and Poisson equations for the mode potential, including drag between species.

Figure 10 shows our first measurements of TG mode damping rates in the temperature regime where drag damping dominates. The new theory predictions (solid lines) are compared to experiments (dots) and theory keeping only classical collisions (dashed lines) with three different species compositions and total densities. Compositions are accurately measured using the TCS method described in Sec. 3a. In all three plasmas the observed TG damping is greater than predicted by classical theory, and is in good agreement with theory that keeps long-range collisions, with no adjustable parameters. Note that when \( T \) is small enough that the plasma becomes strongly-magnetized, i.e. when \( r_s \ll e^2/T \), the classical slowing rate vanishes, since collisional velocity scattering is suppressed by the cyclotron adiabatic invariant [97]; then
only long range collisions can describe the drag damping. At larger temperatures, bounce-harmonic Landau damping begins to play a role (Sec. 3b), while at very low temperatures the plasma becomes strongly-coupled and viscous damping may become important.

In future work we propose to study the related process of parallel velocity diffusion of a particle distribution. An initially non-Maxwellian parallel velocity distribution will relax through collisional diffusion back to a Maxwellian at an enhanced rate due to long-range collisions. Such measurements have been made in neutral plasmas in the regime \( r_c \gg \lambda_D \), \([141]\) but when \( r_c < \lambda_D \) new effects associated with long-range collisions occur. For example, for a single species plasma, long-range 1D collisions do not affect the relaxation of the distribution (in theory) while 3D classical collisions do. This is because, for 1D collisions, two particles of the same species merely exchange parallel velocities, and this does not change the parallel distribution function for that species. In contrast, in a multiple species system, there is enhanced velocity diffusion from 1D long-range collisions, but only due to collisions between different species.

Most importantly, velocity diffusion enters into the evaluation of separatrix dissipation (Sec. 5). On IV, the collisional relaxation of the distribution functions of the three Mg\(^+\) isotopes can be observed directly using our current laser diagnostic setup, and we propose to measure this relaxation to compare to theory that includes long-range collisions between species. On CamV, controlled separatrix experiments will be performed in the cold electron (\( T \sim 25-500 \text{ meV} \)) regime where enhanced slowing is predicted.

**4c. Long-Range Collisional Transport in Strongly-Coupled Ion Plasmas**

We propose to extend our measurements of transport coefficients into the strongly-coupled regime \( \Gamma = e^2 / a T > 1 \), where \( a = (3/4 \pi n)^{1/3} \) is the interparticle spacing (Wigner-Seitz radius). On the IV machine, the strongly-coupled regime is accessed by laser-cooling: \( \Gamma = 0.3(n/10^7 \text{cm}^{-3})^{1/3} / (T/1 \text{K}) \) so \( T \leq 0.3 \text{K} \) is required. We propose to measure test particle diffusion both along and across the magnetic field in the strongly-coupled regime. The methods used previously in our group to make similar measurements will be adapted to the low-temperature strong-coupling regime.

Test particle diffusion will be measured by spin-tagging a group of ions \([142]\) and observing their subsequent diffusive spreading. Since spin tagging and laser cooling cannot be simultaneously performed on the same species, we will laser-cool one Mg\(^+\) isotope, allowing it to sympathetically cool other species through collisions, and spin-tag a different Mg\(^+\) isotope using our second laser, aligned along the plasma axis to tag particles at a given radius. We will then probe the evolution of the tagged species radial density as the tagged ions diffuse radially, as was done in the weakly-coupled regime previously \([124]\). This will allow first measurements of cross-magnetic field test-particle diffusion in a strongly-coupled plasma in a strong magnetic field. For diffusion along the magnetic field, we will send the tag beam across the plasma column, tagging a group of ions at one \( z \) location, and then measure the spread of the tagged ions along \( z \) as a decrease in density versus time within the measurement volume (at the same \( z \)-location as the tag beam).

There have been recent simulations of diffusion in strongly coupled plasmas in magnetic fields \([143,144]\), although with \( \Omega_r / \omega_p \leq 5 \) the B-fields were not a strong as those typically used in IV experiments. The simulations observed unusual 1/B scaling of the parallel diffusion, apparently due to caging effects that are mitigated by cross-field motion, and our measurements may be able to test this. However, our plasmas will be both strongly-coupled and strongly-magnetized, with \( r_c \ll e^2 / T \), so classical Cou-
lomb collisions that scatter the cyclotron velocity vector are suppressed and only long-range ExB drift collisions will play a role in the diffusion. This is a regime not fully explored in previous simulations, so we also propose to develop N-body simulations of the diffusion that involve only guiding center ExB drift dynamics and parallel motion, similar to simulation methods used in studies of mesoscopic plasmas [145].

4d. Energy Equipartition in Multispecies Plasmas

There is good agreement between theory and experiment for the collisional relaxation of strongly magnetized single species plasmas [97, 146-150], where $r_c < e^2 / T$. The relaxation is novel because the collisional dynamics is constrained by adiabatic invariants associated with the cyclotron motion. In a recent publication [17], we extend the theory to the case of a two-species plasma, where the charges of the two species have only slightly different mass. This choice was motivated by the fact that our pure ion plasmas include different isotopes of singly-ionized magnesium. The calculation predicts that the collisional relaxation to thermal equilibrium of the two-isotope plasma proceeds on two well separated time scales.

On the time scale of a few collisions, like ions exchange cyclotron action with each other, and the Gibb’s distribution relaxes to the form $\exp[-H / T] - \alpha_1 I_1 - \alpha_2 I_2$, where $H$ is the total Hamiltonian and the $I_i$ is the total cyclotron action for species $i$. The thermodynamic variables are $T_i$, $\alpha_1$ and $\alpha_2$. On a longer time scale, unlike species exchange action, and the Gibbs distribution relaxes to the form $\exp[-H / T] - \alpha (I_1 + I_2)$. On an even longer time scale, the total cyclotron action ceases to be a good adiabatic invariant, and the distribution relaxes to the standard form $\exp[-H / T']$, where $T' = T_1 = T_2$. The paper predicts this sequence of relaxation times. This work was a portion of the Ph.D. thesis of C.Y. Chim. In future work we will experimentally investigate this multi-stage relaxation for the 3 isotopes of Mg$^+$. 

5. Separatrix Dissipation

5a. Chaotic Superbanana Transport

In most magnetically-confined plasmas, magnetic or potential variations produce phase-space separatrices between locally-trapped and passing particles. Then global “field errors” (such as magnetic tilt or toroidal curvature) affect trapped and passing particles differently, because the particles experience different bounce-averaged perturbations, leading to strong gradients in the velocity distribution function at the separatrix. These strong gradients produce a dissipative boundary layer whose width may be limited by collisions to be of order $v^{1/2}$, leading to the so-called $v^{1/2}$ regime of superbana transport, a form of neoclassical transport (NCT)[151-155]. This collisional separatrix dissipation can also stabilize (or destabilize) different types of waves, and was originally proposed as a mechanism to destabilize trapped particle modes in tokamaks[151].

Our theory and experimental efforts have characterized a new “chaotic” plateau regime of superbanana transport that occurs when the separatrix is asymmetrical (“ruffled”), causing collisionless particle orbits to trap and de-trap chaotically[20-22,156-160]. Then the collisional separatrix boundary layer is superceded by a collisionless layer caused by this chaotic scattering across the ruffled separatrix. Note that this chaotic superbanana transport is independent of, and typically much larger than, the transport caused solely by the asymmetrical ruffle by itself[161]; it requires both the ruffle and a global field error.

Our experiments on chaotic NCT involve two main ingredients:

1) An electrostatic barrier $\phi(r, \theta)$ that creates locally trapped and passing particle populations that undergo different dynamics. For chaotic NCT this barrier is ruffled (asymmetric in $\theta$). In our experiments the ruffle is proportional to $\Delta v_n \cos 2(\theta - \Theta_n)$ (Fig. 11) causing particles near the separatrix energy to trap and de-trap as they bounce in $z$ and ExB drift in $\theta$.
2) A global field error or wave depending on $\theta$ as $\cos(\theta - \theta_0)$, that acts differently on trapped and passing particles. In our experiments, this can be a controlled magnetic tilt, a drift (diocotron) wave, a TG wave, or a combination.

The resulting separatrix dissipation affects many processes, causing enhanced damping of several different types of plasma waves\cite{103}, modified nonlinear coupling in three-wave decay of drift (diocotron) waves\cite{21}, and radial plasma expansion and heating\cite{20,22,159,160,162}. Interestingly, the interaction of separatrix ruffles with applied field errors was found to produce either enhanced or reduced radial transport, depending on the orientation $\alpha = \theta_0 - \theta_a$ of the field error with respect to the ruffle (Fig. 12).

Previous theory work in the fusion community assumes an orientation such that the ruffle reduces the transport, (via an assumed “stellarator symmetry” equivalent to $\alpha = 0$ \cite{163,164,160} but our theory and experiments show that more general separatrix asymmetries produce enhanced rather than reduced plasma loss. For low-collisionality fusion plasmas, this chaotic $\nu$-independent transport may supercede $\nu^{1/2}$ and $\nu^n$ transport processes (Fig. 13).

We believe this is important for systems where field errors and separatrix asymmetries are not well-controlled (as is the case in many fusion plasma systems \cite{165,166}). We note that waves or turbulent fluctuations can also ruffle a separatrix\cite{162}, and we intend to characterize more examples of collisional, chaotic, and wave-induced separatrix dissipation and transport.

5b. Separatrix Dissipation from Axial Sloshing

We propose to study separatrix dissipation in an even simpler geometry, where the dissipation arises from a slow, nearly adiabatic, $\theta$-symmetric axial “sloshing” flow of the plasma through an applied “squeeze” potential barrier. Experimentally, applied end-confinement voltages make the plasma ends move in phase (Fig. 14), so particles trapped on the left or right side of the squeeze potential are heated or cooled adiabatically, but passing particles are nearly unaffected because the overall plasma length is unchanged. (More precisely, passing particles tend to change their density to cancel out the density changes produced by the trapped particles, so as to Debyeshield out the axial electric field produced by the plasma motion). This causes strong gradients in the velocity distribution at the separatrix between trapped and passing particles. The flow is then reversed and repeated at a low frequency $f_{\text{slosh}} \ll f_B$, so that Landau-resonance effects are negligible.

Preliminary IV experiments have measured the plasma heating rate versus $f_{\text{slosh}}$, observing the $\sqrt{f_{\text{slosh}}}$ scaling generally expected for collisional separatrix dissipation (Fig. 15)\cite{160}. However, the measurements show 2-4 times less heating than our initial estimates. On the other hand, applying the opposite “anti-squeeze” voltage (~7 Volts) produces no observable heating effect, establishing that this heating mechanism requires a separatrix (unlike the squeeze-enhanced heating from bounce-harmonic Landau damping, Sec. 3b).

We intend to directly-measure the velocity distribution at the separatrix, using our phase-coherent LIF diagnostic, for quantitative comparison to theory. We will then add $\theta$-asymmetries (ruffles) to the squeeze potential in order to test the predictions of superbanana theory in the chaotic plateau regime. Also, analogous heating rate experiments will be performed on the CamV electron plasma system to establish mass scalings.
6. 2D Plasmas and Crystals
6.a Flux-Driven Algebraic Damping of Diocotron Drift Waves/Kelvin Waves

We have recently discovered [23,25-28] a new variant of spatial Landau damping of exp(imθ) diocotron modes, equivalent to surface waves on 2D vortices [167-171]. The damping occurs when some external process drives a flux of particles (i.e. vorticity) through the wave/rotation resonance (critical) layer $R_m$, defined by $\omega = m f_b(R_m)$. The damping is unusual in that it is algebraic in time rather than exponential. This offers possible explanation for the widely observed rapid symmetrization of vortices: even weak viscosity (which cannot directly dissipate the wave-induced shear) may actually cause symmetrization by driving vorticity through the resonance layer.

The CamV experiment provides quantitative density measurements, allowing accurate calculation of wave-frame potential and critical radius. Fig.16 shows a launched large amplitude $m=2$ diocotron mode, with saturated spatial Landau damping having formed "cat's eyes" around $R_{c2}$ due to density at $R_{c2}$.

Alternately, we can apply a weak magnetic tilt $\varepsilon_B$ which causes a flux of particles to propagate outward, generating a low-density "halo". Even nominally stable $m = 1, 2$ diocotron modes then show strong algebraic damping down to zero amplitude beginning at $t^*$ when the halo flux reaches $R_{cm}$.

That is,
\[
D_m(t) = D_m(t^*) - \gamma_m(t-t^*).
\]

Moreover, the measured damping rate $\gamma_m$ is proportional to the flux of particles through $R_{cm}$ [23,25].

A simple initial theory explained the damping by focusing on the transfer of canonical angular momentum from the wave to resonant particles [25]. In ExB dynamics, the canonical angular momentum of a particle is given by $P_\theta = eB r^2 / 2c$. The outward transport flux moves radially outward to the resonant region, and there the mode rapidly sweeps the particles around the nonlinear cat’s eye orbits to a larger radius. Thus, there is a net transfer of angular momentum from the wave to the particles, and a corresponding damping of the mode.

Although this initial treatment captured the essence of the physics, closer inspection revealed the need for a more rigorous theory. At the leading edge of the halo, where particles are being swept around the cat’s eye orbits, large density gradients develop, so the influence of diffusion in the transport flow cannot be ignored. We recently developed a more rigorous theory for the case of the $m=1$ mode, that includes diffusion effects [27]. Interestingly, the new more rigorous theory yields the same answer for the damping rate as the earlier treatment, provided that the diffusion coefficient is sufficiently small, as is expected to be the case in the experiments. Currently, the more rigorous theory is being extended to the $m=2$ mode.

The more rigorous theory has the advantage of providing an alternate and simple mechanical description of the damping. The $m=1$ mode is special in that an analytic solution for the mode is known for any monotonically decreasing density profile that doesn’t reach the resonant radius. The flux-driven resonant particles produce a dipole electric field that is uniform over the region of the core plasma, and moves the core back toward the axis of the trap, that is, damps the mode.

This analysis led to a simple mechanical interpretation of the general resonant wave-particle interaction [15], namely, the idea that the reaction back on the wave can be understood as arising from action of the bare electric field from the resonant particles on the oscillating non-resonant particles. For the simple case of Landau damping (or growth) of Langmuir waves in a Vlasov plasma, the displacement of non-resonant electrons satisfies an oscillator equation (the wave oscillator), which is driven resonantly by the bare electric field from the resonant electrons, resulting in damping or growth, depending on the relative phase between the oscillator and drive field. Likewise, the spatial Landau damping of diocotron waves can be understood as arising from the action of the electric field from the resonant particles back on the non-resonant particles, although in this case the particles move according to ExB drift dynamics.
Experimentally, this algebraic damping is very effective in suppressing diocotron instabilities, and thus could be useful in fusion applications to suppress drift-type and flute-type instabilities\cite{28}. Diocotron modes in particular have negative energy and can be destabilized by various energy sinks (such as wall resistance \cite{172}). However, the instabilities are suppressed when transport has produced a halo and the algebraic damping mechanism is active. In comparison to exponential growth, the effective damping rate $\hat{D}/D = -\gamma_m/D$ is very large at the outset of an instability when $D$ is small.

Given the broad implications of this unusual algebraic damping process, we intend to pursue further experiments and analysis techniques to obtain quantitative agreement. The theory of algebraic flux-driven Landau damping comprised a portion of the Ph.D. thesis of C.Y. Chim.

6b. 2D Vortex Stripping

While isolated 2D vortices tend toward axisymmetry \cite{173,174} (through spatial Landau damping or viscosity) they can be deformed and/or destroyed by external shear or strain flows. Experiments in viscous fluids such as water, and simulations, have shown that a strained vortex may undergo partial or total destruction as the vorticity is pulled away in thin filaments \cite{175,176} (vortex stripping), but a quantitative understanding of many aspects of these processes is incomplete.

In cooperation with the Surko experimental group, Dubin has studied the relatively simple case of the 2D dynamics when a pure strain flow is rapidly applied to an initially axisymmetric, isolated vortex in an inviscid fluid\cite{29}. The strain velocity field is $\mathbf{v}_s = \varepsilon(x\hat{y} + y\hat{x})$ where $2\varepsilon$ is the strain rate, and the vorticity is $\omega = \nabla \times \mathbf{v}$ with $\mathbf{v}$ the fluid velocity. The dependence of vortex dynamics on the magnitude of $\varepsilon$ and on the initial radial vorticity profile was investigated. Experiments were done using a magnetized pure electron plasma to model an incompressible inviscid 2D fluid \cite{177}, and strain applied via a sectored wall electrode. An advantage of this system is that electron density, which is analogous to fluid vorticity, can be measured directly. Complementary Vortex-In-Cell simulations (Fig.17) were conducted to validate the 2D nature of the experimental results and to extend the parameter range of the studies\cite{174}.

A comparison of the results to a simple elliptical patch model (Kida \cite{178}) shows good agreement for small strain fields where the vortex ellipticity oscillates in the applied strain (Fig. 18); but for larger strains discrepancies appear as the vortex is stripped. More detailed theory and simulations indicate that the fraction of vorticity lost to the wall depends sensitively on the magnitude of the strain and also on the initial radial vorticity profile; this is also observed in the experiments\cite{29}. In future work Prof. Dubin intends to quantify this profile and strain dependence in detail using simulations and theory, in conjunction with experiments by the Surko group.

6c. Ion Crystals Confined to the $z=0$ Plane

In work motivated by discussions with NIST Boulder researchers on quantum entanglement and q-bit implementations, Dubin was able to solve analytically \cite{24} for the planar density per unit area $\sigma(r)$ for a single-species cold plasma confined in the $z=0$ plane in a Penning or Paul trap by an arbitrary external trap
potential, expressed as a multipole expansion. For a system with up to \( M \) applied multipoles, Dubin found an exact finite series solution for the equilibrium plasma areal density versus radius,

\[
\sigma(r) = \sqrt{1 - r^2 / R_p^2} \sum_{n=0}^{M/2-1} \sigma_{2n} r^{2n},
\]

where the plasma radius \( R_p \) and the coefficients \( \sigma_{2n} \) are determined analytically in terms of the applied multipole strengths and the number of trapped ions. Using this solution for the density, the combination of low-order multipoles that produces the most uniform planar plasma crystal with fewest defects can be determined. In simulations, the defects in ion crystals confined in these multipole traps were found to be isolated to the circular edges of the crystal, allowing a large defect-free uniform central region that would be more useful for many AMO studies than non-uniform crystals contained in current quadrupolar trap designs, where the \( M=2 \) multipole is dominant. These multipole configurations are being adopted in current AMO experiments [179].

In addition, Dubin has shown that using a rotating wall field that varies as \( \cos 3(\theta - \omega t) \), the 3-fold symmetry of the ion crystal can be matched to the rotating wall field to produce a triangular crystal with no defects (Fig. 19). We expect this type of theory collaboration to continue in the renewal period.

7. Broader Impacts
The PIs give strong emphasis to the teaching and outreach aspects of their research. Graduate students are diverse in gender, culture, and ethnicity. The 31 graduates from this group over the past two decades have gone on to successful careers in science and industry; and some have themselves become educators, training students in their own right. Undergraduate students are excited by and learn from laboratory and simulation projects through department honors projects and internships; several have co-authored journal articles. The PIs have integrated their research into innovative undergraduate classes: Dubin developed a new undergraduate mathematical physics course and textbook; Driscoll has taught an independent-projects-based course focused on embedded microprocessors. Driscoll, Dubin and O’Neil have all been DPP Distinguished Lecturers, traveling to small colleges and minority colleges. Dubin and Anderegg have lectured on basic plasma physics to international graduate students at Les Houches Winter Schools [30,31,33-36], Dubin and O’Neil have co-authored several review articles of interest to the broader scientific community, [32,190-192] and Dubin and Anderegg have served the APS Division of Plasma Physics in several leadership positions, including Secretary/Treasurer and the Program Committee.

Our plasma research has been rich in connections to the wider world of physics. Our pure electron plasma experiments model the 2D ideal flow of neutral fluids, and we have made definitive measurements of Kelvin wave dynamics, 2D vortex merger, and the inviscid decay of 2D turbulence. Surprisingly, the turbulent decay can result in a “vortex crystal”, intriguingly similar to vortex crystals observed in superconductors and superfluids. Our theoretical and experimental work on trapped ion plasmas has strong connections to the atomic physics, time-standards, and ion crystal work at NIST Boulder, as was recognized through a shared Excellence Prize for Dubin and Bollinger. The magnetically confined pure ion plasmas and crystals are a simple laboratory realization of the OCP model of dense, degenerate astrophysical matter such as the interior of a white dwarf. The experimental demonstration of correlation-enhanced close collisions models the long-predicted Salpeter enhancement of fusion rates in dense, degenerate astrophysical matter. The groups at CERN that have successfully produced and trapped anti-hydrogen credit our group for the development of useful techniques to confine, control, and diagnose their anti-proton and positron plasmas. Our theoretical studies of “guiding center drift atoms” and three-body recombination also support the anti-hydrogen projects. Although much of our work advances basic plasma, the work on neoclassical transport tests and advances fusion-relevant physics. We intend to continue making such interdisciplinary connections during the renewal period.