Chaotic transport and damping from $\theta$-ruffled separatrices

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Variations in magnetic or electrostatic confinement fields give rise to trapping separatrices, and neo-classical transport theory analyzes effects from collision-induced separatrix crossings. Experiments and theory have now characterized a broad range of transport and wave damping effects due to “chaotic” separatrix crossings, which occur due to equilibrium plasma rotation across $\theta$-ruffled separatrices, and due to wave-induced separatrix fluctuations.

Most plasma confinement devices have trapping separatrices, arising from variations in magnetic field strength or external potentials. Separately trapped populations of particles may then have substantially different drift orbits, giving rise to large dissipative transport steps when separatrix crossings occur. Neo-Classical Transport (NCT) theory analyzes the particle transport and wave effects arising from collisional separatrix scatterings in a variety of geometries [1–4]; and experimental corroborations have been obtained in some regimes of strong collisions [5, 6].

Recent experiments and theory have now characterized a novel collisionless form of NCT, where “chaotic” separatrices crossings occur due to plasma rotation across $\theta$-ruffled separatrices, or due to wave-induced separatrix fluctuations. This mechanism has previously been taken to be ineffective because of presumed symmetries [7]. The experiments are performed on low-collisionality, strongly magnetized pure electron plasma columns, with trapping separatrices created by applied wall voltages, or by weak magnetic field strength variations as small as $\delta B_z/B_z \sim 10^{-3}$. The particle transport is driven by an overall “error field,” such as a tilt of the magnetic field [8]. For wave damping, the error field is the wave potential itself, and strong damping is observed for both low-frequency drift waves [9] and high frequency plasma waves [10].

Experiments with controlled separatrix ruffles or temporal variations now unambiguously distinguish the chaotic and collisional contributions. For large $B$, we find that chaotic NCT scales with collision rate and magnetic field as $\nu^0 B^{-1}$, whereas collisional NCT scales as $\nu^{1/2} B^{-1/2}$. The high magnetic field minimizes kinetic bounce-rotation resonance effects [11, 12], which typically scale closer to $B^{-2}$.

Theory analyses of ruffled separatrix effects have now been developed from two complementary perspectives. A dynamical bounce-mapping approach characterizes the quasi-steady-state density perturbations, including bounce-resonant effects in regimes of ultra-low collisionality. A second simpler approach [13] assumes random (chaotic) separatrix crossings, connects smoothly with collisional NCT, and agrees with the dynamical approach outside the bounce-resonant regimes.

The pure electron plasma columns utilized here are confined in a cylindrical Penning-Malmberg trap [8]. Electrons are confined radially by a nominally uniform axial magnetic field $0.4 < B < 20$ kG; and are confined axially by voltages $V_c = -100$ V on end cylinders of radius $R_w = 3.5$ cm. The electron columns have length $L_p = 49$ cm, and radial density profile $n(r)$ with central density $n_0 = 1.6 \times 10^7$ cm$^{-3}$ and line density $N_L = \pi R_p^2 n_0 = 6.1 \times 10^7$ cm$^{-1}$. The neutralized charge results in an equilibrium potential energy $\Phi_c(r)$ with $\Phi_{c0} = +28$ eV at $r = 0$ (here, all $\Phi$’s are in energy units). This gives an $E \times B$ drift rotation $f_E(r)$ which decreases monotonically from $f_{E0} = 230$ kHz ($B$/1 kG)$^{-1}$. The electrons have a near-Maxwellian velocity distribution with thermal energy $T \lesssim 1$ eV, giving axial bounce frequency $f_b = \pi/2L_p = 430$ kHz and rigidity $R \equiv f_b/f_{E0} = 2B_\text{kg}$.

An electrostatic trapping barrier $\phi_s$ is created by a “squeeze” wall voltage $V_{s0}$ with adjustable $\theta$-components. This gives interior separatrix energy $\phi_s(r, \theta) = \phi_{s0}(r) + \Delta\phi_m(r) \cos[m(\theta - \theta_m)]$, as shown schematically in Fig. 1a. Here, we focus on $m = 2$ ruffles, created by voltages $\pm \Delta V_m$ applied to four 60$^\circ$ sectors, extending over $\Delta z = 3.8$ cm near the $z = 0$ center. At every radius, low energy particles are trapped in either the left or right end, whereas higher energy untrapped particles transit the entire length.

Particles change from trapped to untrapped (and vice versa) due to collisions, due to drift-rotation across $\theta$-ruffles, or due to temporal fluctuations $\Delta \phi_t$ in the separatrix energy. The electron-electron collisionality of the present experiments is relatively low; collisions acting for a drift-rotation period spread parallel velocities at the separatrix by an energy width $\Delta W_c \equiv T (\nu/2\pi f_E)^{1/2} (\phi_{s0}/T)^{1/2} \approx 25$ meV $B_{\text{kg}}^{1/2}$. The “chaotic” trapping processes will be important when $\Delta \phi_m \gtrsim \Delta W_c$, or when $\Delta \phi_t \gtrsim \Delta W_c$. The pure electron plasma columns utilized here are confined in a cylindrical Penning-Malmberg trap [8], Electrons are confined radially by a nominally uniform axial magnetic field $0.4 < B < 20$ kG; and are confined axially by voltages $V_c = -100$ V on end cylinders of radius $R_w = 3.5$ cm. The electron columns have length $L_p = 49$ cm, and radial density profile $n(r)$ with central density $n_0 = 1.6 \times 10^7$ cm$^{-3}$ and line density $N_L = \pi R_p^2 n_0 = 6.1 \times 10^7$ cm$^{-1}$. The neutralized charge results in an equilibrium potential energy $\Phi_c(r)$ with $\Phi_{c0} = +28$ eV at $r = 0$ (here, all $\Phi$’s are in energy units). This gives an $E \times B$ drift rotation $f_E(r)$ which decreases monotonically from $f_{E0} = 230$ kHz ($B$/1 kG)$^{-1}$. The electrons have a near-Maxwellian velocity distribution with thermal energy $T \lesssim 1$ eV, giving axial bounce frequency $f_b = \pi/2L_p = 430$ kHz and rigidity $R \equiv f_b/f_{E0} = 2B_\text{kg}$.

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1. ASYMMETRY-INDUCED PARTICLE TRANSPORT

Radial particle transport is driven by global “error fields” varying as $e^{i\theta}$; here, we focus on static $\ell = 1$, $z$-anti-symmetric error fields created by a small magnetic tilt (Fig. 1b). The tilt has controlled magnitude $\epsilon_B \equiv B_1/B_2 \lesssim 10^{-3}$, with chosen tilt angle $\theta_B = \tan^{-1}(B_2/B_1)$, i.e. rotated by $\alpha = \theta_B - \theta_m$ relative to the ruffle. This tilt is equivalent to applying wall voltages $V(R_w, \theta, z) = (\epsilon_B z)(2\pi N_L/R_m) \cos(\theta - \theta_B)$, which causes interior Debye-shielded $\ell = 1$ error potentials $\delta \phi_1(r, z)$, with left- and right-end bounce-averages $\delta \phi_L$ and $\delta \phi_R$. Left- and right-end-trapped particles then have different drift orbits, giving neoclassical radial diffusion coefficient

$$D_r(r) = F_M(\phi_{st}) \left\{ \frac{\Delta W_c D_{cA} + \Delta \phi_m D_{mA} \sin^2 \ell \alpha}{4} \right\},$$

where $F_M$ is the Maxwellian distribution of energies, and $2\ell/m \in \text{Integers}$ [13]. The collisional bounce-Averaged coefficient is $D_{cA} \approx \pi[1 - \exp(-(y/7.1)^{5/6})]$, $y \equiv \Delta W_c/\Delta \phi_m$; and the $m$-ruffle coefficient is $D_{mA} \approx 4(1 - 2.15 \tanh(y/6))$. Both arise from the $z$-anti-symmetry of the error field.

The full radial flux then has both mobility and diffusive contributions, as

$$\Gamma_r = \mu(\partial \phi_c/\partial r) - D_r(\partial N_0/\partial r),$$

with $\mu = D_r N_0/T$ and $N_0 \equiv \int d \phi_r n$. We diagnose the bulk expansion rate

$$\nu_{(r^2)'}(r) \equiv \frac{d}{dt} \langle r^2 \rangle/\langle r^2 \rangle = \int rdr r^2 \frac{1}{r} \frac{\partial}{\partial r} r \Gamma_r/\langle r^2 \rangle.$$

Fortunately, $\nu_{(r^2)'}$ can be accurately and readily obtained from the frequency $f_20$ of a small-amplitude $\ell = 2, k = 0$ dicotatron mode, as $\nu_{(r^2)'} = -f_20/f_{20}$. This follows from $f_{20} \propto \langle n \rangle = N_L/(\langle r^2 \rangle$ with $N_L$ constant; and it has been verified to ±2% by camera images of plasma evolutions.

Figure 2 is a plot of measured expansion rate $\nu_{(r^2)'}$ versus magnetic tilt angle $\theta_B$, for various applied wall ruffle strengths $\Delta V_m$. The ruffled-induced NCT shows an unambiguous $\sin^2 \alpha$ dependence on relative angle $\alpha$, with magnitude proportional to $\Delta V_m$; and varying $\theta_m$ in steps of $\pi/2$ (not shown) verifies the dependence on relative angle only. At $r = 1 \text{ cm}$ where the separatrix energy is about $1.5 \text{ eV}$, an applied $\Delta V_m = 1 \text{ Volt}$ gives separatrix ruffles $\Delta \phi_m \sim 30 \text{ meV}$, compared to collisional width $\Delta W_c \sim 50 \text{ meV}$.

The distinctive $\sin^2 \alpha$ signature, together with control of $\Delta V_m$ and $\epsilon_B$, enables identification of transport processes as

$$\nu_{(r^2)'}^{(\text{expt})} = C_{cA} \epsilon_B^2 + C_m \epsilon_B \Delta V_m \sin^2 \alpha + C_{cK1} \epsilon_B^2 + C_{cK2} \Delta V_m + \nu_{(r^2)'}^{(\text{bkg})}.$$ 

Here, $C_{cA}$ and $C_{mA}$ represent the radial integrals of Eqn. (1) and (2); $C_{cK1}$ and $C_{cK2}$ represent collisional Kinetic (bounce-resonant) transport driven by $\epsilon_B^2$ or $\Delta V_m^2$ as $z$-dependent “error” fields; and $\nu_{(r^2)'}^{(\text{bkg})}$ arises from uncontrolled background tilts, separatrices, and ruffles. For dimensional simplicity, $\epsilon_B = \epsilon_B/(1 \text{ mRad})$ and $\Delta V_m \equiv \Delta V_m/(1 \text{ Volt})$.

$C_{mA}$ is readily obtained from the $\sin^2 \alpha$ dependence as in Fig. 2, and varying $\epsilon_B$ gives the expected $\epsilon_B^2$ dependence. Data taken with $\epsilon_B = 0$ shows a $\nu_{(r^2)'}^{(\text{bkg})}$ offset and a parabolic dependence on applied $\Delta V_m$, giving $C_{cK2}$. Varying $\epsilon_B$ then selects $C_{cA}$ and $C_{cK1}$; these are distinguished by their $B$-scaling (discussed next), and by the fact that the $z$-anti-symmetric bounce-averages in $C_{cA}$ require the separatrix, whereas the kinetic $C_{cK1}$ depends only weakly on the applied squeeze voltage. In Fig. 2, $C_{cK2} \sim 0.03$, giving elevated $\sin^2 \alpha$ minima for large $\Delta V_m$; the depressed minima for $\Delta V_m = 0.3$ are from ruffle-suppression of $D_{cA}$; and $\nu_{(r^2)'}^{(\text{bkg})} \sim 0.007$.

Figure 3 shows the measured transport rates $C_{mA}$, $C_{cA}$ and $C_{cK1}$ versus magnetic field with empirical scalings (dashed), compared to theory (lines). At high $B$, the chaotic and collisional separatrix transport processes agree closely with theory, scaling as $B^{-1}$ and $B^{-3/2}$ respectively. Here the comparison is limited by temperature uncertainty, sensitivity to edge density gradients, and induced modification of $F_M(\phi_{\text{mapp}})$. At low $B$, the kinetic transport labeled $C_{cK1}$ is observed to depend strongly on field ($\sim B^{-2/7}$), but no simple power-law is expected as bounce-rotation resonances become dominant. Prior scaling experiments have been confused by the presence of uncontrolled separatrices and ruffles, and by overlapping transport regimes [8].

Similar enhanced transport is observed when there are temporal variations in the separatrix energy. Figure 4 illustrates the immediate increase in radial expansion rate induced when white noise ($V_{\text{RMS}} = 200 \text{ mV}$, $f_{E0} < f < 20 \text{ MHz}$) is applied to the $\theta$-symmetric squeeze ring, causing chaotic trapped-passing transitions. The $3\times$ increase in $(d/dt)\langle r^2 \rangle$ observed here is consistent with a collisional separatrix layer $\Delta W_c \sim 85 \text{ meV}$ fluctuating by $\Delta \phi_1 \sim 200 \text{ meV}$. Here, any noise or wave-induced fluctuations which change particle kinetic energies relative to the separatrix energy would be equally effective in enhancing transport.

2. WAVE DAMPING

Wave damping due to chaotic and collisional separatrix dissipation is observed for both negative-energy $E \times B$
drift waves and for positive-energy plasma waves. Here, the wave-potential is the “error field” driving transport (Fig. 1b), and the wave frequency enters the generalization of Eq. (1). Most thoroughly studied is the “Trapped Particle Diocotron Mode” [3, 14] where end-trapped particles at large radii experience $z$-anti-symmetric $E \times B$ drifts, while untrapped interior particles provide partial Debye shielding.

Prior TPD damping analysis [3] solved for the thin collisional boundary layer at the separatrix, as is standard in NCT [1, 2]. This gave quantitative agreement with the experimental observations of $\gamma_{1a} \propto B^{-1/2}$ for large $B$; but the enhanced damping observed at lower $B$, scaling as $\gamma_{1a} \propto B^{-1}$, was not understood.

Experiments and theory now quantify the $B^{-1}$ scaling of TPD damping as due to $\theta$-ruffles on the separatrix. Figure 5a shows the measured TPD damping rate $\gamma_{1a}$ versus strength $\Delta V_m$ of an applied $m = 2$ separatrix ruffle, for two magnetic fields. For $\Delta V_m = 0$, the damping is mostly due to collisions; for larger $\Delta V_m$, the damping increases linearly with $\Delta V_m$ as expected for chaotic NCT. Here, $\gamma_{1a} \propto B$ is plotted, so the identical slopes at $B = 0.4$ and $B = 3$ represent the $B^{-1}$ scaling characteristic of chaotic separatrix processes, analogous to the $\Delta V_m$ terms of Eq. (1).

In contrast, the collisional $\Delta V_m = 0$ intercepts scale as $B^{-1/2}$, and therefore differ by $(3/0.4)^{1/2} = 2.7$. The solid curves of Fig. 5a are the absolute predictions of the probabilistic theory approach, including non-local effects on $n(r, z)$ equilibria [15].

Similar enhanced damping is seen when a separate wave ruffles the separatrix. Figure 5b shows TPD damping in the presence of a separate $m = 2$, $k_z = 0$ diocotron mode with frequency $f_{20} \approx f_{E0}$ and controlled quadrupole amplitude $Q$, with $Q = \Delta/R_0$ for uniform density out to radius $R = R_0 + \Delta \cos \theta$ [9]. The diocotron mode creates an $m = 2$ potential $\phi_m(r, z, t)$ at all $z$, which is smallest at the $z = 0$ separatrix, inducing chaotic separatrix crossings proportional to $Q$. The solid line segments show a $Q = 0$ intercept predicted by collisional NCT [3], and a ruffle-induced enhancement predicted by the bounce-mapping theory, in good agreement with the measurements.

The resonant version of this same wave-wave interaction was partially explored in experiments [9] where a large-amplitude $m = 2$ diocotron mode “pump” decays into an exponentially growing $\ell = 1$ TPD mode, with parameters tuned so as to obtain $f_{1a} = f_{20}/2$. The separatrix causes a well-understood conservative mode coupling, exponential instability, and late-time energy sloshing; and the previously puzzling dissipation is now quantitatively understood as TPD damping from a phase-locked $m = 2$ ruffle caused by the diocotron mode.

High frequency plasma waves are also strongly damped by separatrix dissipation, independent of Landau damping effects, but critically dependent on the characteristics of the separatrix. Figure 6 shows the measured damping rate $\gamma_{11}$ for an $\ell = 1$, $k = 1\pi/L_p$ plasma wave with $f_{11} = 1.2$ MHz. This is a large amplitude wave in a “BGK state” of strong wave-particle trapping. With no applied electrostatic squeeze, damping at rate $\gamma_{11}^{(M)} \sim -1 \times 10^3$/sec is observed, due to a naturally occurring magnetic separatrix $\delta B_z/B_z \sim 10^{-3}$ peaking near $z = 0$. This magnetic separatrix often dominates background transport also, and removing the separatrix reduces $\nu_{(bb)}^{(bkg)}$ by up to 5x.

In Fig. 6, adding a ramped positive (anti-) squeeze wall voltage has no effect on $\gamma_{11}$; but a negative squeeze ramped to -3 Volts immediately and proportionately increases $\gamma_{11}$, to a maximum of $\gamma_{11}^{(V)} = -8 \times 10^3$/sec. Here, Zakharov-Karpman [16] collisional damping predicts negligible damping, at a rate $\gamma_{11}^{(ZK)} \sim -20$/sec. We also note that excitation of a separate $\ell = 0$ plasma wave to even moderate amplitude immediately increases $\gamma_{11}$ several fold, due to $\Delta \phi_m$ in the effective energy of the separatrix.

In summary, we find that chaotic separatrix crossings arising from plasma rotation across ruffled separatrices or from temporal separatrix fluctuations cause dissipative effects which may dominate the simpler collisional effects. Overall, there are a broad range of particle transport, wave damping (or instability), and wave couplings which are only beginning to be experimentally characterized.

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FIG. 1: (Color online) a) $\theta$-symmetric end confinement and central separatrix potentials, modified by an $\ell = 1$ $z$-antisymmetric error field, and an $m = 2$ ruffle on the separatrix; b) $z$-dependence of 3 error fields.

FIG. 2: (Color online) Measured expansion rate $\nu_{\langle r^2 \rangle}$, showing chaotic NCT varying as $\sin^2 \alpha$, and $\alpha$-independent collisional transport.
FIG. 3: (Color online) Measured transport rates $C$ versus $B$, with empirical scalings. Solid lines are theory predictions.

FIG. 4: (Color online) Enhanced expansion rate during two bursts of 200 mV (RMS) noise applied to a 6 V electrostatic separatrix.

FIG. 5: (Color online) a) (bottom, right scales) TPDM damping rate $\gamma_{1a}$ times $B$ versus applied $\Delta V_m$ for $B = 0.4$ and 3.0 kG. b) (top, left scales) TPDM damping $\gamma_{1a}B$ versus amplitude $Q$ of an excited diocotron mode.
FIG. 6: (Color online) Separatrix damping of a Langmuir wave: $\gamma_{11}^{(M)}$ from a weak magnetic mirror (red), and $\gamma_{11}^{(V_{eq})}$ (black) due to a time-varying negative $V_{eq}$ (blue).