ION-INDUCED INSTABILITY OF DIOCOTRON MODES IN ELECTRON PLASMAS
MODELLING CURVATURE-DRIVEN FLUTE MODES

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We study ion-induced instability of flute-like \((k_z \approx 0)\) diocotron modes in pure electron plasmas confined in a cylindrical Penning-Malmberg trap. In the absence of positive ion contamination, the low \(m_0\) diocotron modes are either neutrally stable (for \(m_0 = 1\)) or weakly damped (for \(m_0 = 2,3\ldots\)) by Landau resonance on electrons co-rotating with the diocotron waves. By adding a small fraction (< 1%) of positive ions into a double-well confinement configuration, we observe exponential instability of low \(m_0\) diocotron modes. The growth rates \(\gamma_m\) are directly proportional to the overall ion fraction, \(N_i/N_e\), and proportional to an effective charge separation of electrons and ions in the periodic wave perturbation.

I. INTRODUCTION

Curvature-driven flute modes have been a problem for magnetic confinement from the beginning of fusion research\(^1\). The charge separation produced by a curving magnetic field causes rapid loss of particles from \(\mathbf{E} \times \mathbf{B}\) drifts. Magnetically confined pure electron plasmas are naturally free from this problem, until a small fraction of positive ions is added. Then, the ions trapped in a double-well confinement configuration may destabilize the electron plasma diocotron modes, analogous to the curvature driven destabilization of neutral plasma flute modes.

It is of general interest to see the onset of the flute (interchange) instability in the transition between these two \((N_i = 0\) and \(N_i \approx N_e\)) extremely different regimes. Also, this strong instability may have important implications for a variety of experiments, such as the double-well traps to overlap the positron and anti-proton clouds for production of anti-hydrogen\(^2,3\).

In our experiments we use a Penning-Malmberg trap\(^4\) in a double-well (nested) configuration (see Fig. 1). The electron column of length \(L_e \leq 50\) cm is contained inside a stack of hollow conducting cylinders of radius \(R_w = 3.5\) cm, which reside in an ultrahigh vacuum with residual pressure \(\approx 10^{11}\) Torr. The inner end cylinders are negatively biased (−100 V) with respect to the electron plasma potential \((\phi_e \approx −30\) V) to axially confine electrons, while the outer end cylinders are grounded or biased positively (+40 V) to axially confine ions. A strong axial magnetic field \((B \approx 20\) kG) ensures radial confinement both for the electrons and ions.

Typical electron columns have central density \(n_e \approx 1.5\times10^7\) cm\(^{-3}\) over a bell-shaped radial profile with a characteristic radius \(R_p \approx 1.2\) cm, giving line density \(N_e \approx n_e \pi R_p^2 \approx 7\times10^7\) cm\(^{-3}\). The unneutralized electron charge results in an \(\mathbf{E} \times \mathbf{B}\) rotation of the column at frequency \(f_B = cE/2\pi vB \sim (0.1\) MHz\)(2kG/B). These pure electron plasmas have exceptional confinement properties and can be contained for hours\(^5\).

The positive ion population \((\text{H}_2^+)\) is created either by controlled external injection of an ion beam into cold \((T_e \approx 0.5\)–1 eV) electron columns; or by ionization of the background gas within r.f.-heated \((T_e \approx 7\)–9 eV) electron plasmas.

The flute instability in neutral plasmas is driven by the drift charge separation due to curving \(\mathbf{B}\)-field; here the instability is driven predominantly by the bounce-averaged \(\mathbf{E} \times \mathbf{B}\) drift charge separation due to the difference in confinement regions for the electrons and ions in the double-well trap. Let us to consider a cartoon of this process based on as yet unjustified assumption that the ion motion becomes stochastic on the diocotron time scale \(\tau_d = 1/\nu_i\). In Figure 2, for visual simplicity we consider the \(m_0 = 1\) diocotron perturbation (off-axis shift) of amplitude \(d\). However, the very same basic reasoning is valid for any \(m_0\) diocotron modes. In the shown double-well configuration the trapped ions spend a part \(\tau_{end}\) of their overall bounce time \(\tau_{bnc}\) outside the trapped electron column, where they lag in phase behind the electron diocotron density perturbation by a typical phase shift \(\theta_\phi = 2\pi\tau_{end}/\tau_{bnc}\), which is accumulated over a full diocotron period.
Note that the growth rate $\gamma_m$ for the trapped-ion-induced instability of diocotron modes is also functionally similar to the growth rate of the passing-ion-induced instability of diocotron modes, which may be expressed as $\gamma_p = (N_i/N_e) f_p [1 - \cos(2\pi f_p/\omega)]$. Here, $f_p \approx 1/T_{\text{bun}}$ is the ion single pass time through the electron column, and $f_p$ is the electron column rotation frequency. This passing-ion-induced instability gives us the background growth rate in case of modulated ion trapping experiments.

II. SOME EXPERIMENTAL RESULTS

The growth rates $\gamma_m(t)$ for the $k_z = 0$, $m_0 = 1, 2, 3, \ldots$ diocotron modes are measured by digitizing the amplification $A_m(t)$ of corresponding wall potentials induced by the diocotron oscillations at the azimuthally sectored electrodes. These amplitudes are verified (and calibrated) by taking corresponding $(r, \theta)$ moments of 2D-density distribution of the dumped electron column from a CCD camera diagnostic. In the experiments, we usually keep the mode amplitudes, $A_m(t) \approx A_m(0)/R_w$ (scaled by the wall radius $R_w$), small enough ($d_m \approx 0.01$) to have nonlinear effects well constrained. The trapped ion fraction $N_i(t)/N_e$ is obtained simultaneously with $\gamma_m(t)$ by measuring the relative frequency change of the $m_0 = 1$ diocotron mode $\Delta f_i(t)/f_i$ during modulated ion beam trapping, since fractional neutralization gives frequency change as $\Delta f_i = (N_i/N_e) f_i$.

Figure 3 demonstrates the 2-decade exponential growth of the $m_0 = 1$ diocotron mode in linear regime. Here, the ion fraction $N_i/N_e \approx 5 \times 10^4$, and $f_i \approx 4 \text{ kHz}$ ($B = 7 \text{ kG}$). Note that $f_i \approx f_0(R_p/R_w)^2$. Modes with higher $m_0$-mode wave numbers show faster initial growth in accord with their higher frequencies $f_m \approx [m_0 - 1 + (R_p/R_w)^2] f_0$, i.e., $\gamma_m / \gamma_i \approx f_m / f_i$, but then they rapidly approach nonlinear saturation due to spatial Landau damping in the radial edge of the plasma column.

While frequencies of diocotron mode are obvious function of $B (f_m \propto 1/B)$, their growth rates, normalized by the ion production (injection current) rate per electron, $\nu_i$, show no dependence on $B$ (see Fig. 4). This is consistent with the measured linear $B$-scaling of the active ion lifetime $\tau_f = N_i/N_e \nu_i$ in electron column (Fig. 5), which also indicates an $E \times B$ drift nature of the ion radial losses. For our plasma and trap parameters it takes from hundreds to thousands of diocotron cycles for an average ion to reach the radial edge of electron plasma, and details of this process are still ill understood.

Besides the rather trivial linear dependence of $\gamma_m$ on the ion fraction, we have also qualitatively verified the $\sin^2$-dependence, $\gamma_1 / \Delta f_1 = 2 \sin^2(\pi \nu_{\text{end}}/\nu_{\text{bun}})$, on the effective charge separation. The most simple double-well configuration (adjacent cylinders used to confine electrons and ions) typically gives us $\pi \nu_{\text{end}}/\nu_{\text{bun}} \approx 0.1$, which should result into the slope $\gamma_1 / \Delta f_1 \approx 0.2$. Figure 6
shows this is quite reasonable estimate. Increasing $\tau_{\text{end}}$ by
adding cylinders to the ion part of the trap and simultaneously
shrinking the length of the electron column, we first have observed a gradual increase of the
$\gamma_1/\Delta f_1$ slope up to $1.6$ ($L_e \approx L_{\text{end}}$), which then been
followed by a decrease down again to about 0.2 at
$\tau_{\text{end}}/\tau_{\text{bec}} \approx 0.9$. More quantitative comparison is
complicated by the coarseness of confinement length
steps and the ill-defined dependence $\tau_{\text{end}}$ on the ion
energy and axial potential distribution.

Fig. 3. Exponential growth of the $m_0 = 1$ diocotron mode
in linear regime ($d_1 \leq d_{\text{cr}} << 1$).

Fig. 4. Growth rate per injected ion is independent of the
magnetic field. Big error bars are due to large uncertainty
in measurements of the ion injection rate at pA level.

Fig. 5. Active lifetime $\tau_f$ of an average ion as a linear
function of the magnetic field. The lifetime is measured
from exponential saturation of fractional neutralization
$\Delta f_2(t)$ after the beginning of ion injection,
$\Delta f_2(t) = -f_2(0)\nu_1/\tau_f[1 - \exp(-t/\tau_f)]$.

Fig. 6. Growth rate as a linear function of fractional
neutralization ($\gamma_1/\Delta f_1 = 0.22, f_1 = 3889$ Hz at $N_f = 0$).

III. CONCLUSIONS

Instabilities of diocotron modes are commonly
observed when small ion fractions are added to pure
electron plasmas\textsuperscript{10-16}. However, historically these
instabilities were first thought to be driven by the
difference in rotation frequencies of the two species
caused by inertial (mass difference) effects. This is
analogous to the centrifugal MHD-instabilities in neutral
plasmas, which are usually less dangerous than magnetic
curvature driven (charge difference) modes if they do not
belong to a narrow parameter range (resonant effects).

Here, we find that the ion-induced instability is
driven by an entirely different physical mechanism.
Broadly, the different bounce-averaged azimuthal drifts of electrons and ions tend to polarize the diocotron mode density perturbations, thereby developing instability similar to the classical flute MHD-instability of neutral plasmas confined in non-uniform magnetic fields. The major factor that distinguishes charged plasma behavior in our experiment from neutral plasma confinement is that this charge separation mainly comes from the two species sampling different radial electric fields at the plasma column ends (prevalent importance of this effect for double-well traps was first mentioned by Pasquini and Fajans\textsuperscript{15,16}). However, it allows us an easy control over the effective drift separation of oppositely charge particles in the wave perturbation, by simply varying the ratio of the ion end transit time $\tau_{\text{end}}$ (by adjusting the end length $L_{\text{end}}$ or the potentials) to the total bounce time $\tau_{\text{bnc}}$.

As a result, the measured trapped-ion-induced growth rates show a rather simple dependence

$$\gamma_m = \left( \frac{N_i}{N_e} \right) f_m \times \left[ 1 - \cos \left( 2\pi \frac{\tau_{\text{end}}}{\tau_{\text{bnc}}} \right) \right]$$

verified in a broad range of relevant plasma and trap parameters. Having just the linear reduction factor, $N_i/N_e$, this strong exponential instability has no threshold on the smallness of the ion fraction, and without paying proper attention it may lead to significant rate of particle radial transport and losses in double-well confinement configurations.

**ACKNOWLEDGMENTS**

This work was supported by National Science Foundation Grant No. PHY0354979.

**REFERENCES**


