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Transport Toward Thermal Equilibrium in a Pure Electron Plasma*

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Abstract A new theory of cross-magnetic field transport due to like-particle collisions is presented. The new theory supercedes the traditional theory in the parameter regime where the Debye length is large compared to the Larmor radius ($\lambda_D \gg r_L$); the flux predicted by the new theory exceeds that predicted by the traditional theory by a factor of $O(\lambda_D^2/r_L^2) \gg 1$. Furthermore, transport due to lightly damped waves can enhance the flux in certain cases. The elementary step in the new transport process is due to the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift of a particle guiding center which occurs during a binary collision. Previous discussions of like-particle transport considered only the regime $r_L \gg \lambda_D$; in this case the step in the guiding center position is due to collisional scattering of the velocity vector. The regime $r_L \ll \lambda_D$ is standard for magnetically confined pure electron plasmas. Experiments are discussed in which transport toward thermal equilibrium is measured in such plasmas. The measured flux agrees with the new theory rather than the traditional theory.

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I. Introduction

This talk describes a new theory of cross-magnetic field transport due to like-particle interactions and the consequences of this theory for the transport toward thermal equilibrium of a pure electron plasma. A pure electron plasma consists of an unneutralized collection of electrons contained by electric and magnetic fields. It is well known that under ideal conditions such plasmas can achieve confined thermal equilibrium states.[1] In order to explain experiments now underway at UCSD,[2] it is necessary to consider a hitherto unexamined regime in the theory of like-particle transport: the regime in which $r_L \ll \lambda_D$, where r_L is the electron Larmor radius and λ_D is the electron Debye length. Traditional theories[3,4] of transport due to like-particle collisions were intended to describe ion-ion interactions in neutral plasmas and were formulated for the regime $r_L \gg \lambda_D$. The transport mechanism considered in the new theory yields a particle flux which greatly exceeds that predicted by the traditional theory in the regime $r_L \ll \lambda_D$.

The ratio of the particle flux in the new theory to that in the traditional theory will be seen to be of $O(\lambda_D^2/r_L^2)$ under the assumption that electrons interact only via a Debye-shielded potential. Furthermore, an even larger flux is possible if collective effects are taken into account. However, in the regime of current experiments, theory indicates that the influence of collective effects on the transport is probably negligible.

Experiments have measured the transport toward thermal equilibrium, and these experiments will be discussed in the last part of the talk. Briefly, the scaling of the measured flux with the magnetic field strength follows the new theory rather than the traditional theory, and the magnitude of the flux agrees with the new theory, not the traditional theory. While these conclusions bode well for the new theory, the experiments are still in a preliminary stage and much work remains to be done.

II. Theory of Like-Particle Transport

Consider a pure electron plasma in slab geometry bounded by two conducting plates at $x = 0$ and $x = x_0$. A constant magnetic field in the z-direction permeates the plasma. The plasma is assumed to be homogeneous in the y and z directions; density and potential gradients are assumed to be functions only of x (see Fig. 1). (In this talk we take the temperature T to be constant for the sake of simplicity.)

Let us first consider the flux from a fluid dynamic, or macroscopic, perspective. The electric field and pressure gradient produce a fluid drift given by

$$\underset{\sim}{v}(x) = -\frac{1}{m\Omega_e}\left[eE + \frac{1}{n}\frac{d}{dx}(nkT)\right]\hat{y} \quad , \tag{1}$$

where Ω_e is the electron cyclotron frequency, $n(x)$ is the density, $E(x)$ the electric field and m is the electron mass.

Because of viscosity, the shear in this fluid drift gives rise to a force density $\underset{\sim}{F}(x)$ which may be written as

$$\underset{\sim}{F}(x) = \frac{d}{dx}\, nm\nu\Delta x^2 \frac{dv}{dx}\;\hat{y} \quad , \tag{2}$$

where ν and Δx are frequency and length scales which characterize the collisional dynamics and the combination $nm\nu(\Delta x)^2$ is the viscosity. The particle flux in the x-direction is due to the $\underset{\sim}{F}\times\underset{\sim}{B}$ drift and is given by

$$\Gamma_x(x) = -\frac{1}{m\Omega_e}\;\underset{\sim}{F}\times\hat{z}\circ\hat{x} = \frac{d}{dx}\; n\nu\Delta x^2\; r_L^2\;\frac{d}{dx}\left[\frac{eE}{kT} + \frac{1}{nkT}\frac{d}{dx}(nkT)\right] . \tag{3}$$

The quantities ν and Δx must be determined through a microscopic description of the plasma. In both the traditional theory and the new theory, ν can be identified as the collision frequency $[\text{i.e.}, \; \nu \sim O(\omega_p/n\lambda_D^3)]$. However, the traditional theory and the new theory yield different predictions for Δx (i.e., $\Delta x \simeq r_L$ and λ_D, respectively).

III. Scaling in the Traditional Theory

The traditional theory[3,4] was intended to describe transport due to ion-ion collisions in neutral plasmas and implicitly assumes that $\lambda_D \ll r_L$. The range of the interaction between two particles is assumed to be much smaller than the Larmor radius; effectively, collisions are treated as interactions at a point. During a collision, the position of a particle does not change, but the guiding center changes abruptly as a result of collisional scattering of the velocity vector (see Fig. 2).

Consider two guiding centers at $\underset{\sim}{r}_{1G.C.}$ and $\underset{\sim}{r}_{2G.C.}$, separated in the x-direction by Δx. The electrons associated with these guiding centers are at $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ and the equation for the x-component of the guiding center position is

$$x_{G.C.} = x - v_y/\Omega_e \quad ,$$

where v_y is the y-component of the particle velocity. As the electrons approach within a Debye-length of one another, their velocity vectors scatter producing a change in guiding center position;

$$\Delta x_{G.C.} = -\Delta v_y/\Omega_e \quad .$$

However, conservation of momentum implies that

$$\Delta x_{1_{G.C.}} + \Delta x_{2_{G.C.}} = \frac{1}{\Omega_e} \Delta [v_{1y} + v_{2y}] = 0$$

so the guiding centers step in equal and opposite directions. Thus, if $\Delta x = 0$, there is no net flux in the x-direction from the collision; that is, the flux goes to zero as Δx approaches zero. In fact, as we stated in the last section and as we will soon prove, $\Gamma_x \propto \Delta x^2$, for small Δx. Furthermore, we see from this analysis that $\Delta x \sim r_L$ since guiding centers further apart than $\sim 2r_L$ cannot interact (recall that we assume $\lambda_D \ll r_L$; we also neglect collective effects here). Substituting $\Delta x \sim r_L$ in Eq. (3) implies that the flux in the traditional theory scales as

$$\Gamma_x^{\text{traditional}} \propto \nu r_L^4 \quad . \tag{4}$$

We now compare this scaling with that obtained in the new theory of transport.

IV. The New Theory of e-e Transport

In this section we determine the scaling of the flux due to e-e collisions in the regime where $r_L \ll \lambda_D$. This is the typical regime for magnetically confined pure electron plasmas. In this case, most electrons collide via a guiding center drift mechanism which is quite different than the point collisions of the traditional theory. This mechanism is illustrated in Fig. 3. Assume, as before, that collective effects may be neglected so that electrons interact via a Debye-shielded potential. Then as electrons spiral along adjacent magnetic field lines and approach within a Debye length of one another, the interaction electric field causes the electrons to $\underset{\sim}{E} \times \underset{\sim}{B}$ drift; it is this drift which produces the basic transport step in the new theory. The drifts are still in equal and opposite directions but now the length scale of the interaction is of $O(\lambda_D)$. Thus the flux in the new theory scales like

$$\Gamma_x^{\text{new}} \sim \nu r_L^2 \lambda_D^2 \quad , \tag{5}$$

which is $O(\lambda_D^2/r_L^2)$ larger than in the traditional theory.

In a recent paper,[5] O'Neil calculated the flux according to the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift theory of collisions assuming that the electrons interact only via a Debye-shielded potential. He finds

$$\Gamma_x = \frac{d}{dx} n(x) K(x) \frac{d}{dx} \left[\frac{1}{n}\frac{dn}{dx} + \frac{eE}{kT}\right] \tag{6}$$

where

$$K(x) = \frac{1}{96\,\pi^{3/2}} \frac{\omega_p}{n\lambda_D^3} \log\left(\frac{\Delta v}{v_t}\right) r_L^2 \lambda_D^2 \quad .$$

The form of K(x) does indeed follow the scaling of Eq. (5). The logarithmic term in the collision frequency is due to the effect of neighboring electrons with nearly the same velocities. These electrons interact for long times and hence take relatively large steps. The minimum value of $\Delta v/v_t$ may be set by various mechanisms. One mechanism is decorrelation due to collisions, which gives $\Delta v/v_t \sim (n\lambda_D^3)^{-1/3}$. Another possible value for Δv is set by the difference in $\tilde{E}\times B$ drift velocities of electrons a Debye length apart; $\Delta v/v_t \sim (c/B\omega_p)(d\tilde{E}/dx)$.

V. Collective Effects: A Scaling Argument

O'Neil's treatment of the new theory cannot be considered rigorous, since he made the ad-hoc assumption that two electrons interact only through a Debye-shielded potential. Here, we relax this assumption and recognize that two electrons can interact over a distance which is much larger than a Debye length through the emission and absorption of lightly damped waves. The damping rate (and emission rate) for a given mode is inversely related to the interaction length characterizing the mode [i.e., $\nu_L = v_g/\Delta x$, where v_g is the group velocity and ν_L the Landau damping rate]. For modes characterized by the interaction length $\Delta x \gg \lambda_D$, we will find the effective interaction frequency

$$\nu \sim f \frac{\omega_p}{n\lambda_p^3} \frac{\lambda_D}{\Delta x} \quad , \tag{7}$$

where $f \sim 10^{-3}$ is a dimensionless constant which enters through a sum over modes. From Eq. (3), it follows that

$$\Gamma^{\text{waves}} \sim f \frac{\omega_p}{n\lambda_D^3} r_L^2 \Delta x \, \lambda_D \tag{8}$$

Comparing this to Eq. (5) we see that transport due to waves will be important provided that

$$f \gtrsim \lambda_D/\Delta x \quad . \tag{9}$$

The largest possible value of Δx is the slab thickness x_0, so if the plasma is large enough or λ_D small enough, the scaling argument indicates that wave transport can be important. We will see that in the regime of the transport experiments the inequality is not satisfied implying that transport due to waves is probably negligible. However, in experiments on cryogenic plasmas which are underway, wave transport may be important.

Of course, for Δx sufficiently large, the microscopic theory does not reduce to fluid dynamic equations which incorporate the notion of a local viscosity. The local theory, as expressed in Eq. (3), implicitly assumes that Δx is smaller than the spatial scale length characterizing the variation of the density and electric field.

VI. Flux Calculation Using the B.B.G.K.Y. Hierarchy

This section provides a flux calculation which incorporates the general physical ideas presented in the last section. Using a reduced guiding center form of the B.B.G.K.Y. hierarchy, we recover the result of O'Neil in one limit and obtain corrections due to collective effects in another limit. We use a guiding center form of the hierarchy since large impact parameter collisions dominate the particle transport. The hierarchy is formed on a reduced phase space incorporating only parallel velocity U and guiding center position $\underset{\sim}{x}$.

From the first equation of this B.B.G.K.Y. hierarchy (or simply from inspection) one can see that the electron flux in the x-direction is given by the expression

$$\Gamma_x = \frac{-e}{m\Omega_e} \int d^3x_2 \, \frac{\partial \phi_{12}}{\partial y_1} \, g_{12} \, dU_1 \, dU_2 \ , \tag{10}$$

where ϕ_{12} is the unshielded interparticle potential and g_{12} is the two-particle correlation function. In order to calculate the flux, we must calculate the correlation function. The equation for g_{12} can be obtained from the B.B.G.K.Y. hierarchy. By neglecting 3-particle correlations we obtain

$$\left[\frac{\partial}{\partial t} + \mathcal{L}_1 + \mathcal{L}_2\right] g_{12} = \frac{-e}{m} \frac{\partial \phi_{12}}{\partial z_1} \left[\frac{\partial}{\partial U_1} - \frac{\partial}{\partial U_2}\right] f_1 f_2 + \frac{e}{m\Omega_e} \frac{\partial \phi_{12}}{\partial y_1} \left[\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right] f_1 f_2, \tag{11}$$

where $f_1 = f(x_1, U_1, t)$ is the one-particle distribution function,

$$\mathcal{L}_1 h = U_1 \frac{\partial h}{\partial z_1} - \frac{E(x_1)}{B} c \frac{\partial h}{\partial y_1} - \frac{c}{B} \frac{\partial \phi}{\partial y_1} \frac{\partial f_1}{\partial x_1} + \frac{e}{m} \frac{\partial \phi}{\partial x_1} \frac{\partial f_1}{\partial U_1} \ ,$$

and

$$\nabla^2 \phi = 4\pi e \int h dU_1 \ .$$

We will solve this equation for g_{12}, assuming that g evolves towards its equilibrium form at a rate large compared to the rate of change of f (the so-called 'Bogoliubov ansatz'). We therefore neglect the time dependence of f when solving for g. We first write g as

$$g_{12} = g_{12}^{(0)} + g_{12}^{(1)}$$

where $g_{12}^{(0)}$ is the solution of Eq. (11) in the $B \to \infty$ limit. Since the flux goes to zero as $B \to \infty$, this limit describes a plasma which will achieve thermal equilibrium along field lines, but can still support arbitrary density gradients across the field in equilibrium. We assume $g_{12}^{(0)}$ reaches its equilibrium form on a scale much faster than $g_{12}^{(1)}$, i.e. that thermal equilibrium is quickly set up along the field lines. This assumption relies on the existence of small impact parameter collisions which thermalize the velocity distribution. These collisions do not contribute substantially to transport and are not included directly in the guiding center analysis; they enter the theory only implicitly. Solving for $g_{12}^{(0)}$ under this assumption yields the result

$$g_{12}^{(0)} = f_M(U_1)\, f_M(U_2)\, G_{12}^{(0)} \quad ,$$

where $G_{12}^{(0)}$ is independent of velocity, f_M is a Maxwellian at temperature T, and $G_{12}^{(0)}$ satisfies

$$G_{12}^{(0)} - \frac{e}{T} \int d^3x_3\, n(x_3)\; \phi_{13}\, G_{23}^{(0)} - \frac{e}{T}\, \phi_{12} = 0 \quad .$$

The equation for $g_{12}^{(1)}$ is then

$$\left[\frac{\partial}{\partial t} + \mathcal{L}_1 + \mathcal{L}_2\right] g_{12}^{(1)} = S_{12} \quad , \tag{12}$$

where

$$S_{12} = \frac{\partial G_{12}^{(0)}}{\partial y_1}\, [v(x_1) - v(x_2)] \quad .$$

It is then not difficult[6] to show that the solution for $g_{12}^{(1)}$ is given by a product of the Green's functions for the two linear Vlasov operators $\mathcal{L}_1$ and $\mathcal{L}_2$; when $g_{12}^{(1)}$ is integrated over velocities we then find

$$\int g_{12}^{(1)} dU_1\, dU_2 = \int_0^t dt \int d^3\bar{x}_1\, d^3\bar{x}_2\, \delta n_{1\bar{1}}(t)\, \delta n_{2\bar{2}}(t)\, S_{\bar{1}\bar{2}} \tag{13}$$

where $\delta n_{1\bar{1}}$ is the density response function (Green's function integrated over velocity) for the linear Vlasov equation:

$$\delta n_{1\bar{1}} = \int \delta f_{1\bar{1}}\, dU_1 \tag{14a}$$

where

$$\left[\frac{\partial}{\partial t} + \mathcal{L}_1\right] \delta f_{1\bar{1}} = \delta(x_1 - \bar{x}_1)\, \delta(t)\, f_M(U_1). \tag{14b}$$

We now use Eq. (13) to determine the flux in two interesting limits.

VII. Rederiving O'Neil's Result, Plus Some 'Singular Behavior'

It is possible to retrieve Eq. (6) by assuming that the density response function is highly localized, i.e. the response of the plasma to a perturbation propagates only a short distance compared to the density and potential gradient scales. In this case the density reponse is the same as that for an infinite homogeneous plasma, and we may then schematically write the Fourier transform with respect to x_1 of $\delta n_{1\overline{1}}$ as

$$\delta\bar{n} \sim \frac{1}{D(\bar{x}_1)} \quad , \tag{15}$$

where $D(\bar{x}_1)$ is the 'local' dielectric function for a homogeneous plasma with properties determined by the density and potentials at $\bar{x}_1$. Since Eq. (13) shows that $g_{12}^{(1)}$ is proportional to a product of density response functions it should then not be surprising that

$$g_{12}^{(1)} \sim \frac{1}{|D|^2} \quad . \tag{16}$$

This equation contains the origin of the 'singular behavior' to which the title of this subsection refers, but before we get to that I want to write down the result for the flux that one obtains by making this local approximation. It is

$$\Gamma_x = \frac{2\omega_p}{n_0\lambda_D^3}\, n_0 r_L^2 \lambda_d^2 \int \frac{d\bar{\omega}\, d^3\bar{k}}{(2\pi)^4}\, \frac{\bar{k}_x^2\,\bar{k}_y^2}{\bar{k}^4\,|D(x_1)|^2}\, \frac{|Z(\xi)|^2}{\bar{k}_z^2(1+\bar{k}^2)^2}\, \frac{d^2}{dx_1^2}\, \frac{eE}{kT} \quad , \tag{17}$$

where $\underset{\sim}{\bar{k}} = \underset{\sim}{k}\lambda_D$, $\bar{\omega} = \omega/\omega_p$, $\xi = \bar{\omega}/\sqrt{2}\,|\bar{k}_z|$ and Z is the plasma dispersion function. For the sake of simplicity, we have taken the density to be constant in determining Eq. (17), although there is (at least theoretically) no problem in keeping the density arbitrary. We see that the flux does indeed depend on $|D|^{-2}$, but the point is that if we now substitute for D the Debye shielding dielectric,

$$D = 1 + 1/\bar{k}^2 \quad , \tag{18}$$

we regain O'Neil's result, after performing the ω, k integrals. This should not be surprising since by using Eq. (18) we assume that the plasma acts only to Debye-shield the interparticle interaction, which is the same assumption made by O'Neil.

However, if we naively substitute the full plasma dielectric into Eq. (17) and attempt to perform the integrations, we run into a problem. The dielectric function exhibits zeros at frequencies $\omega(k) = \omega_r - i\gamma$ corresponding to the dispersion relation of waves which propagate in the plasma. This implies $|D|^{-2}$ has singularities at the wave frequencies and it is not difficult to show that, for small γ,

$$\int \frac{d\omega}{|D|^2} \sim \frac{1}{\gamma} \left| \frac{dD}{d\omega_r} \right|^{-2} , \tag{19}$$

which becomes very large as the wave damping becomes small. This divergence is, in fact, an indication that processes involving lightly damped waves may be important for the transport. However, there is clearly something wrong because Eq. (19) implies that waves which are completely undamped provide an infinite contribution to the transport. In fact such waves should not cause transport at all since by definition they do not interact with the electrons and can neither be launched nor absorbed in the inter-electron interaction process.

The resolution of this paradox lies in the assumption behind Eq. (15), i.e. that the density response is highly localized. This is clearly violated if there are lightly damped waves excited by a perturbation. In fact, since we are dealing with a system of finite thickness x_0, normal modes will, in general, be excited by a perturbation. In the next section, we will calculate the contribution of the normal modes to the transport and compare this contribution to the non-resonant contribution obtained by O'Neil.

VIII. Transport Due to Modes

As we saw in the last section, the local approximation for δn given schematically by Eq. (15) fails to describe the plasma response when lightly damped waves are excited. Such waves set up modes in the plasma slab and a more natural description of the excitation is as a sum of eigenmodes.

Fourier transforming Eq. (14) with respect to $y_1 - y_2$, $z_1 - z_2$ and t, we may write the Fourier-transformed density response function $\delta n_{1\bar{1}} = \delta n(x_1, x_{\bar{1}}, k_y, k_z, \omega)$ as

$$\delta\bar{n}_{1\bar{1}} = \frac{-i}{\sqrt{2}\, v_t |k_z|} Z(\xi_{\bar{1}}) \sum_n \left(1 + \frac{k^2(x_1)}{\lambda_n} \right) \psi_n(x_1)\, \psi_n(\bar{x}_1)$$

where $\xi_1 = \left(\omega + \frac{k_y E(x_1) c}{B} \right) \Big/ \sqrt{2}\, v_t |k_z|$, $k^2(x_1) = k_x^2(x_1) + k_y^2 + k_z^2$, and ψ_n satisfies the eigenmode equation

$$\left[\frac{d^2}{dx_1^2} + k_x^2(x_1) + \lambda_n \right] \psi_n(x_1) = 0 , \quad \psi_n(0) = \psi_n(x_0) = 0 ,$$

and

$$k_x^2(x_1) = - \frac{(1 + \xi_1 Z(\xi_1))}{\lambda_D^2(x_1)} + \frac{d \ln n}{dx_1} \frac{k_y \omega_p^2}{\sqrt{2}\, v_t |k_z| \Omega_e} Z(\xi_1) - k_y^2 - k_z^2 .$$

The eigenvalue $\lambda_n(\omega, k_y, k_z)$ can be thought of as a global dielectric function; that is, $\lambda_n \to 0$ at the wave frequency $\omega = \omega_n(k_y, k_z) - i\gamma_n(k_y, k_z)$ corresponding to the eigenmode ψ_n. By using the time-asymptotic form of the inverse transform for $g_{12}^{(1)}$, we obtain

$$\Gamma_x = \frac{4\pi e^2}{m\Omega_e}\int \frac{d\omega\, dk_y\, dk_z}{(2\pi)^3} \frac{k_y^2}{2v_t^2 k_z^2} \sum_{n,\bar{n}} \frac{\psi_n(x_1)\psi_{\bar{n}}^*(x_1)}{\lambda_{\bar{n}}^*}\left(1 + \frac{k^2(x_1)}{\lambda_n}\right) I_{n\bar{n}} ,$$

where

$$I_{n\bar{n}} = \int_0^{x_0} d\bar{x}_1 d\bar{x}_2\, \bar{G}_{12}^{(0)}\, n(\bar{x}_1) n(\bar{x}_2)[v(\bar{x}_1) - v(\bar{x}_2)]\psi_n(\bar{x}_1)\psi_{\bar{n}}^*(\bar{x}_2) ,$$

$$\text{and} \qquad \bar{G}_{12}^{(0)} = \bar{G}^{(0)}(x_1, x_2, \omega, k_y, k_z)$$

is the Fourier transform of $G_{12}^{(0)}$.

Note that one term in the flux is proportional to a factor of the form $1/\lambda_n\lambda_n^*$. Since $\lambda_n(\omega, k_y, k_z)$ vanishes for $\omega = \omega_n - i\gamma_n$, it follows that λ_n is small for ω near ω_n provided that γ_n is small, that is, that mode n is weakly damped. There is a large contribution to the flux from two weakly damped modes with nearly equal frequencies, since λ_n and $\lambda_{\bar{n}}$ are both small in the same frequency range. However, it is important to note that the contribution from $n = \bar{n}$ is small since $\text{Re}(I_{nn}) = 0$. If the damping and frequency difference $\Delta\omega = \omega_n - \omega_{\bar{n}}$ are small, it is not difficult to show that

$$\int \frac{d\omega}{\lambda_n \lambda_{\bar{n}}^*} = \frac{\pi}{\frac{\partial \lambda_n}{\partial \omega_n}\frac{\partial \lambda_{\bar{n}}^*}{\partial \omega_{\bar{n}}}} \frac{2\bar{\gamma} - i\Delta\omega}{\Delta\omega^2 + 4\bar{\gamma}^2} ,$$

where $\bar{\gamma} = (\gamma_n + \gamma_{\bar{n}})/2$. In the limit that the system becomes homogeneous and infinite, the frequency difference $\Delta\omega$ between modes can approach zero and the frequency integral again diverges as γ^{-1}, as in the local approximation of Sec. VII. Inhomogeneities and the finite system size now limit the value of $\Delta\omega$ and thus limit the size of the integral. This effect is in qualitative agreement with the argument presented in Sec. V in which we set the maximum value of Δx at x_0, thus limiting the size of the flux due to collective effects.

In order to make a quantitative estimate of the flux due to collective effects, we must determine eigenfrequencies and eigenmodes, which is in general a rather difficult (though well-defined) calculation. We therefore have limited ourselves to a special case in which we take the density to be constant and the electric field sufficiently slowly varying so that eigenmodes for a homogeneous plasma slab are good approximations. Substituting

$$\psi_n = \sqrt{\frac{2}{x_0}} \sin \frac{n\pi x}{x_0}$$

and

$$\lambda_n = \frac{1 + \xi Z(\xi)}{\lambda_D^2} + \frac{n^2 \pi^2}{x_0^2} + k_y^2 + k_z^2$$

we then find that, in the limit of small λ_D/x_0, the flux due to collective effects is

$$\Gamma_x^{\text{collective}} = C \sum_{m=1}^{\infty} I_m \frac{m\pi}{x_0} N_m \cos \frac{m\pi x}{x_0} ,$$

where we have Fourier-transformed the gradient in the electric field;

$$\frac{1}{kT} \frac{dE}{dx} = \sum_{m=1}^{\infty} N_m \sin \frac{m\pi x}{x_0} + N_0 ,$$

$$C = \frac{\omega_p}{n_0 \lambda_D^3} n_0 r_L^2 \lambda_d^2$$

and I_m is a wavenumber integral similar to that in Eq. (17) but which is finite as wave damping approaches zero. It is found that I_m scales, for small λ_D/x_0, like

$$I_m \sim \bar{k}^{*6} \frac{x_0}{m\lambda_D} ,$$

where $\bar{k}^*$ is the wavenumber (in units of the Debye length) of plasma modes with a damping length equal to x_0; $\bar{k}^* \sim 0.2\text{-}0.3$. This wavenumber enters because modes with damping which is too great have a small Δx, while modes with damping that is too small do not interact with the electrons. In other words, modes with wavenumber $\bar{k}^*$ dominate the collective transport. Thus, flux due to collective effects scales like

$$\Gamma_x^{\text{collective}} \sim \bar{k}^{*6} \frac{\omega_p}{n_0 \lambda_D^3} r_L^2 \lambda_D x_0 \tag{21}$$

Comparing this with Eq. (8), we see that the coefficient f is given by $f \sim \bar{k}^{*6}$. Furthermore, Eq. (9), the criterion for transport due to waves being important, becomes

$$\bar{k}^{*6} \gtrsim \lambda_D/x_0 , \tag{22}$$

which implies $\lambda_D/x_0 \leq 10^{-3}$. However, in the current transport experiments $\lambda_D/x_0 \sim 0.1$ where for x_0 we use the radius of the plasma. Thus, this theory indicates that transport due to waves is unimportant and the result of O'Neil is essentially correct for these experiments. However, we must redo the calculation for more realistic plasmas in cylindrical geometry before a final conclusion can be drawn. Experiments on cryogenic plasmas with much smaller values of λ_D/x_0 are currently underway, and in these experiments transport due to waves may play a more important role.

IX. Experimental Results

We now turn to a discussion of the results of experiments conducted at UCSD which have measured the transport to thermal equilibrium in a confined pure electron plasma column. The confinement geometry is shown schematically in Fig. 4. Electron plasmas are created by heating a filament, which emits electrons. The electrons stream along the magnetic field and are captured by applying negative biases to the cylinders A and C. Typical densities are in the range of $n \sim 10^7 \text{cm}^{-3}$, temperatures $T \sim 1$ eV, with magnetic fields $B \sim 100$ Gauss. The radius of the plasma r_p is typically about 2 cm, and $r_L \sim 0.25$ mm while $\lambda_D \sim 2.5$ mm, so we are within the range of validity of O'Neil's theory.

Density and temperature measurements as a function of radius are made by grounding cylinder C, which allows the electrons to stream along field lines out the end of the machine. Electrons on a particular field line pass through a collimator and the number of electrons and their temperature are measured. Radial profiles are then obtained by moving the collimator and repeating the experiment several times.

It is well-known that such plasmas can achieve confined thermal equilibrium states.[1] Lack of space prevents a complete discussion of these states; suffice to say that the existence of such states depends on cylindrical symmetry of the system as well as the fact that particles with only one sign of charge are confined. In a given magnetic and external electrostatic field, the thermal equilibrium may be characterized by a temperature T, total number of particles N and a rotation frequency ω (ω is a constant since in thermal equilibrium the plasma must be a rigid rotor). Thermal equilibrium density profiles for a given ω and T and different N are shown in Fig. 5.

When the plasma is initially created, it is not in thermal equilibrium, i.e., T and ω are not constants. However, the profile evolves with time towards thermal equilibrium. Such evolution has been observed experimentally (see Fig. 6). The density profile, which is originally quite irregular, relaxes to a curve resembling those shown in Fig. 5, and the rotation frequency becomes independent of radius. Furthermore, the amount of time required for the plasma to come to equilibrium, τ, may be measured as a function of external parameters, and compared to the predictions of theory. For instance, the traditional theory of transport predicts that, everything else held constant, τ scales like B^4, while the new theory predicts that τ scales like B^2 (see Eqs. (4) and (5)). In unpublished work, Driscoll has estimated τ experimentally; the results are plotted in Fig. 7. While the estimation is crude and can be considerably improved, τ clearly agrees with the B^2 scaling of the new theory and disagrees with the B^4 scaling of the traditional theory. Furthermore, the magnitude of the measured τ is in agreement with O'Neil's theory but is orders of magnitude out of agreement with the traditional theory.

X. Conclusion

A new theory of cross-magnetic field transport due to like-particle collisions was developed and experimental tests of the theory were described. The theory supercedes traditional theories of transport in the regime $r_L \ll \lambda_D$ and experimental results in this regime are consistent with the new theory rather than the traditional theory. The effects of waves on the transport process were considered, and it was found that waves are not important in the operating regime of current transport experiments, but may be more important for cryogenic plasma experiments which are currently underway.

Acknowledgments

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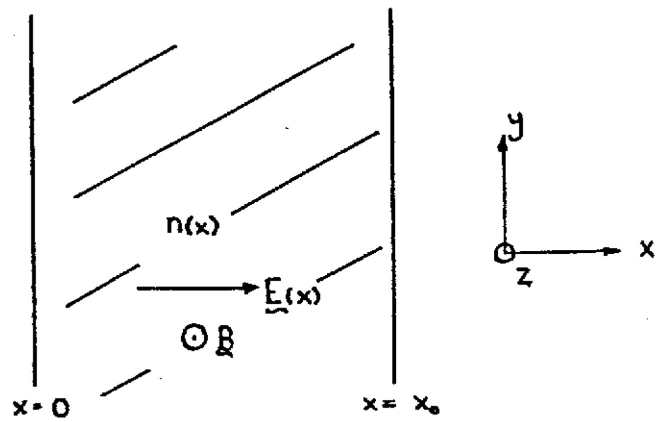

Fig. 1. Configuration of plasma slab for theory of sections II-VIII.

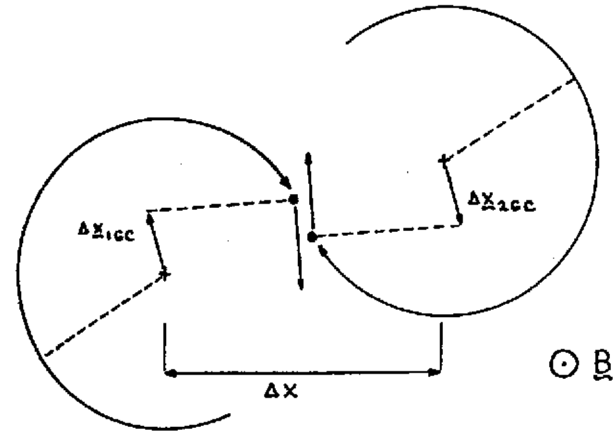

Fig. 2. A collision in the traditional theory. Electron and initial guiding center positions are given by dots and crosses, respectively.

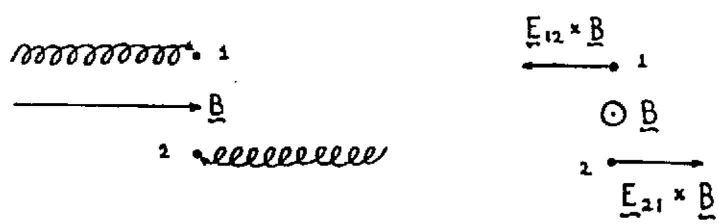

Fig. 3. A collision in the new theory. A) Electrons, labelled 1 and 2, stream along field lines. B) End on view: the electrons $\underset{\sim}{E} \times \underset{\sim}{B}$ drift in their interaction electric field.

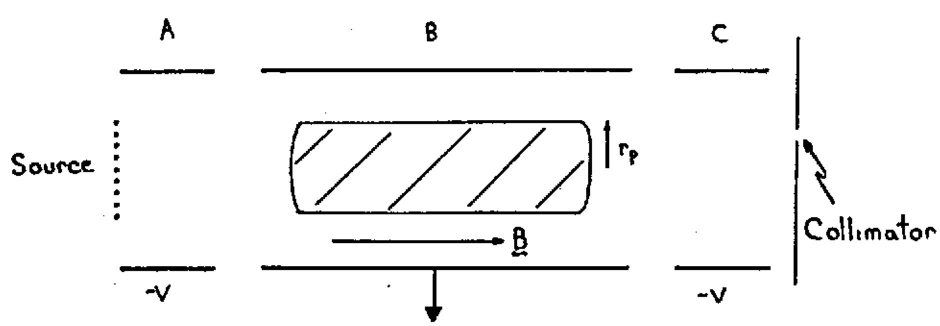

Fig. 4. Schematic diagram of plasma containment device.

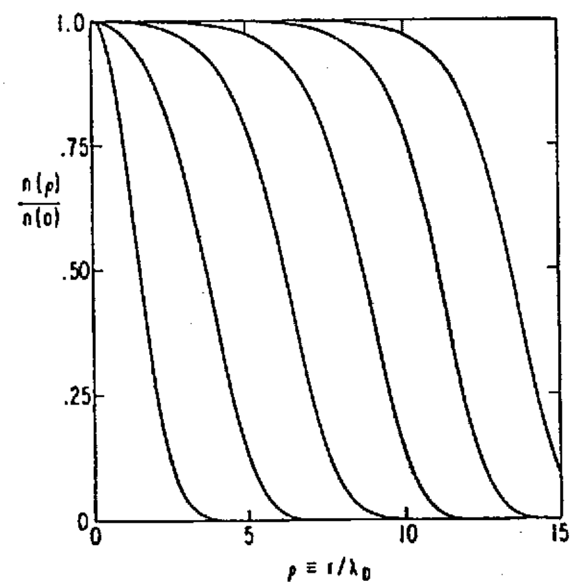

Fig. 5. Theoretical prediction of thermal equilibrium density profiles for given ω, T and varying N.

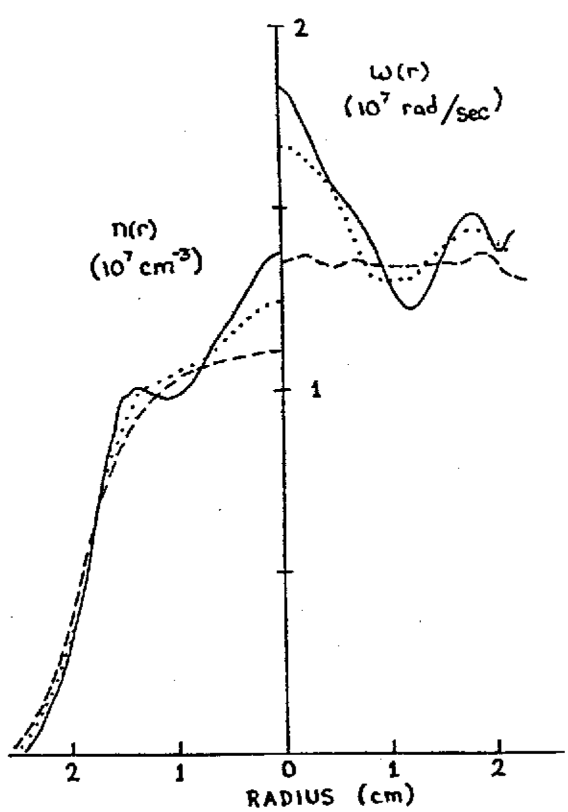

Fig. 6. Density and rotation frequency profiles at 3 different times. Solid curves: immediately after plasma formation (t = 0). Dotted curves: t = .5 sec. Dashed curves: t = 2 sec.

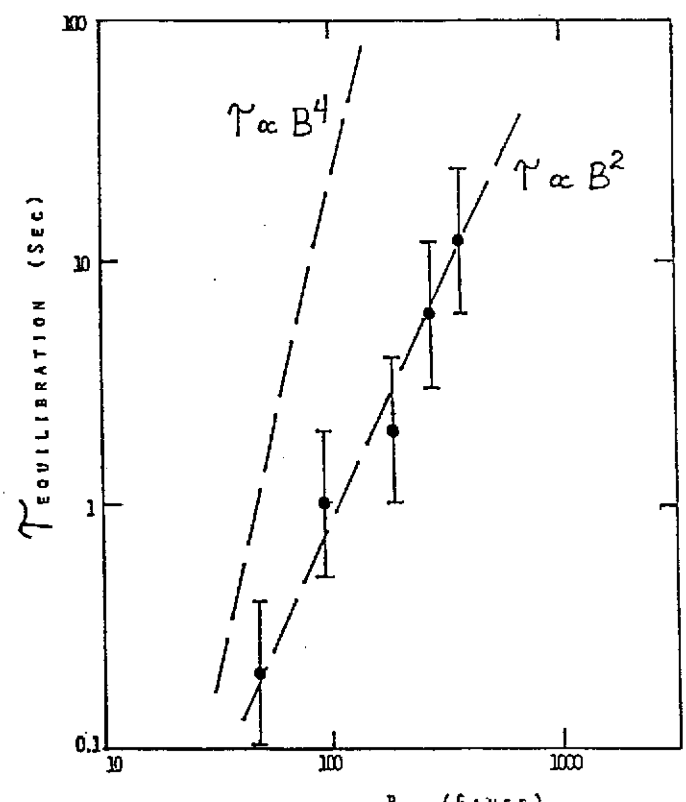

Fig. 7. Scaling of equilibration time τ with magnetic field. Dots are experimental points. Dashed lines show that τ scales like B^2 rather than B^4.