EXPERIMENTS ON NEOCLASSICAL ASYMMETRIC SUPERBANANA ROLLER TRANSPORT IN ELECTRON PLASMA

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Ripples in magnetic or electrostatic confinement fields give rise to trapping separatrices, and the conventional neoclassical ripple transport describes phenomena coming from the collisional trapping/detrapping of particles in the helical ripple wells. Our experiments and novel theory have now characterized a new kind of neoclassical transport processes arising from chaotic (collisionless) separatrix crossings, which occur due to equilibrium $E \times B$ plasma rotation along poloidally asymmetric (ruffled) separatrices, and due to wave-induced separatrix modulations.

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1. INTRODUCTION

Neoclassical transport due to axial asymmetries is ubiquitous in magnetic fusion plasma confinement. These plasmas typically have several helically trapped particle (superbanana) populations, either by natural design (stellarators) or due to the finite number of toroidal field coils (tokamaks), partitioned by separatrices from one another and from toroidally trapped (banana) particle trajectories. The drift orbits for particles trapped in the two toroidally separate regions are displaced radially from one another, leading to the conventional neoclassical superbanana ripple transport as particles collisionally change (at rate $\nu$) from helically trapped to toroidally trapped and back. Neoclassical ripple transport theory analyzes the particle transport and wave effects arising from collisional scattering across the ripple separatrix in a variety of geometries [1-4], and experimental corroboration has been obtained in some regimes of strong collisions [5, 6].

This situation is dramatically modified when the ripple separatrix is itself poloidally asymmetric (ruffled), or when it fluctuates due to waves in the plasma. In such a case the particles see a time-varying separatrix barrier, and without needing collisions they can chaotically transit between helically and toroidally trapped populations. This mechanism can substantially modify particle transport in low collisionality regimes associated with fusion plasmas, though it has previously been considered to be ineffective due to a presumed symmetry of such transitions [7].

In our experiments with $\theta$-ruffled separatrices these chaotic crossings lead to considerably enhanced (or noticeably suppressed, for that matter) neoclassical ripple transport, depending on the relative phase $\alpha$ between the toroidal tilt and the separatrix ruffle asymmetries. The experiments utilize externally controlled electrostatic ruffles or fluctuations on the separatrix, and can thus identify the novel chaotic neoclassical ripple transport scaling as $\nu B^2 \sin^2 \alpha$, and thus distinct from collisional neoclassical ripple transport scaling as $\nu B^2 / \Omega$.

2. EXPERIMENTAL SETUP

The experiments utilize a cylindrical Penning-Malmberg trap to confine quiescent, low-collisionality pure electron plasmas [8-10]. Electrons are confined radially by a nominally uniform axial magnetic field $0.04 < B < 2$ T; and are confined axially by voltages $V_e = -100$ V on end cylinders of radius $R_e = 0.035$ m. The electron columns have length $L_e = 0.49$ m, and radial density profile $n(r)$ with central density $n_0 \approx 1.6 \times 10^{13}$ m$^{-3}$ and line density $N_e = n_0 R_e \approx 6.1 \times 10^{20}$ m$^{-4}$. The unneutralized charge results in an equilibrium potential energy $\Phi_e(r)$ with $\Phi_{e0} \approx +28$ eV at $r = 0$ (here, all $\Phi$’s are in energy units). This gives an $E \times B$ rotation frequency $f_\Omega(r)$ which decreases monotonically from $f_{\Omega0} \approx 230$ kHz$/(B/1$ kG)$^{-1}$. The bulk electrons have a near-Maxwellian velocity distribution with thermal energy $T \sim 1$ eV, giving axial bounce frequency $f_\delta \approx 430$ kHz and rigidity parameter $R_g \equiv f_\delta / f_e \approx 2 R_w$. Fig.1. Schematic of electron plasma with tilt $\epsilon_B$ and a trapping barrier in a cylindrical Penning-Malmberg trap

As a helical ripple substitute, an electrostatic trapping barrier $\phi_{\delta}(r, \theta)$ is created by a “squeeze” wall voltage $V_{sq}$ (see Fig.1) with adjustable $\theta$-sector voltages $\pm \Delta V_w$. This gives controllable interior separatrix energy $\phi_{\delta}(r, \theta) = \phi_{\delta0}(r) + \Delta \phi_n \cos[m(\theta - \theta_0)]$. Here we focus mostly on $m = 2$ ruffles, created by voltages $\pm \Delta V_w$ applied to four 60° sectors, extending over $\Delta \theta = 3.8$ cm near the $\theta = 0$ center. At every radius, low energy particles are trapped in either the left or right end, whereas higher energy passing particles transit the entire length of the column. Ruffles spread the characteristic separatrix energy by $\Delta \phi_n(r) \sim \Delta V_n (r/R_w)$, somewhat reduced by plasma shielding.

Particles change from ripple trapped to passing (and vice versa) due to binary collisions at rate $\nu$, due to drift-rotation across $\theta$-ruffle variations $\Delta \phi_n$, or due to temporal fluctuations $\Delta \phi (t)$ in the separatrix energy. The electron-electron collisionality in the present experiments is
relatively low ($\nu \sim 100$/sec), and collisions acting for a drift-rotation period spread the separatrix by an energy width $\Delta W_e = \nu(3/2\pi)^{1/2}e\psi/(b/1kG)^{1/2}$. Thus, the chaotic (de)trapping processes will be important when $\Delta \phi_m(r) \geq \Delta W_e$, or when $\Delta \phi(t) \geq \Delta W_e$.

We diagnose the bulk expansion rate $\nu_{\nu_{22}}$, defined as

$$V_{\nu_{22}} = \frac{1}{\langle r^2 \rangle} \frac{d}{dt} \langle r^2 \rangle.$$  \hspace{1cm} (1.1)

Fortunately, it can be accurately and readily obtained from the continuous frequency shift $f_2(t)$ of a small amplitude $m = 2$ diocotron mode, as $\nu_{\nu_{22}} = (1/f_2) d f_2/dt$.

The bulk expansion rate $\nu_{\nu_{22}}$ is an integral measure of the full radial flux that includes both mobility and diffusive contributions, both being proportional to the radial diffusion coefficient $D_r(r)$.

### 3. ASYMMETRY-INDUCED TRANSPORT

Radial particle transport is conveniently driven by a global (toroidal) magnetic tilt asymmetry with controlled magnitude $\varepsilon_B \equiv B_z/B_r \lesssim 0.001$ and gradually chosen tilt direction $\theta_B = \tan^{-1}(B_z/B_r)$. This tilt is equivalent to applying $z$-antisymmetric wall voltages $V_w(R,\theta,z) = \varepsilon_B z (2\varepsilon_B R_c/R_w) \cos(\theta - \theta_B)$, which causes interior Debye-shielded $z$-asymmetric potentials $\delta\phi_z(r,z)$. For large $B$ fields, giving rigidity $R_g \gg 1$, simple $z$-bounce-averaged theory suffices to describe the separatrix-induced transport and wave-damping. The tilt-induced $z$-asymmetric error field $\delta\phi_z(r,z)$ has bounce averages $\bar{\delta}\phi_z$ and $\bar{\delta}\phi_z$ for left- and right-end trapped particles near the separatrix energy, with passing particles experiencing zero bounce-average error field. The drift orbits then for left- and right-end superbanana trajectories differ radially by

$$\Delta r = \left(\bar{\delta}\phi_z - \bar{\delta}\phi_z\right)/\bar{\delta}\Phi_e/\bar{\delta}r.$$  \hspace{1cm} (1.2)

Random transitions between trapped and passing populations are caused by collisions (c); by drift rotation along the $\cos(m\theta)$ separatrix ruffles ($m$); and by temporal fluctuations in the separatrix energy ($t$). If the fraction of particles transitioning in a rotation period is $\eta$, the radial diffusion coefficient is expected to be

$$D_r = \eta f_{\nu_{22}} \Delta r^2.$$  \hspace{1cm} (1.3)

For collisions, conventional neoclassical ripple transport gives $\eta_c = \Delta W_e F_M(\phi_m) \propto \nu^{1/2} B^{1/2}$, where $F_M$ is the Maxwellian distribution of energies, whereas ruffle $\eta_m$ and temporal $\eta_t$ are independent of $\nu$ and $B$.

A detailed analysis of random transitions between equal trapping regions driven by rotation across separatrix ruffles gives neoclassical asymmetric superbanana radial diffusion coefficient

$$D_r(r) = \frac{1}{2} F_M(\phi_m) f_e \Delta \phi^2$$

$$\times \left\{ \Delta W_e D_{CA} + \Delta \phi_c D_{2A} \sin^2 \alpha \right\}.$$  \hspace{1cm} (1.4)

Both the collisional bounce-Averaged transport coefficient $D_{CA}$ and the $m = 2$ ruffle coefficient $D_{2A}$ are shown in Fig. 2, calculated in [11] as functions of the normalized ruffle strength $\Delta \phi_c/\Delta W_e$. While the ruffle-induced transport coefficient $D_{2A}$ is nearly independent of

$\Delta \phi_c/\Delta W_e$, the collisional coefficient $D_{CA}$ shows a fast decline as chaotic particle transitions become the dominant ones and smoothly out the discontinuity of $F_M$. In the case of properly aligned asymmetries ($\sin^2 \alpha = 0$), this could enable some suppression of collisional neoclassical ripple transport, until bounce-resonant transport processes become significant.

![Fig. 2. Calculated collisional $D_{CA}$ and ruffle induced $D_{2A}$ coefficients versus the normalized ruffle strength](image)

![Fig. 3. Sketch of split E*B drift orbits near the $m = 2$ ruffled separatrix. a) $\alpha = 0$. b) $\alpha \neq 0$. For the magnetic tilt asymmetry the trapped portions of the orbits are partial circles shifted along the tilt direction](image)

Prior theory [17] considered only $\alpha = 0$ or $\pi$, in which case the phase-dependent part of the diffusion coefficient is zero. The reason for this can be qualitatively understood from Fig.3a, which shows a sketch of split $E*B$ drift orbits near the $m = 2$ ruffled separatrix. From the magnetic tilt asymmetry the trapped portions of the drift orbits are partial circles shifted along the tilt direction. If this direction coincides with the zero phase of separatrix ruffle, the right-left symmetry implies particles transit from trapped to passing and back at the same radius, so the drift orbit is closed and there is no net radial step. However, when $\alpha \neq 0$, the symmetry is broken and particle orbits are trapped and detrapped at different radii, leading to radial steps $\Delta r \neq 0$ (Fig.3b). Of course, for $\alpha = 0$ or $\pi$ the diffusion does not completely vanish; collisional effects not kept in the above analysis yield finite diffusion consistent with the one obtained in [7].
Figure 4 shows the predicted transition from predominantly collisional neoclassical diffusion to the chaotic regime (ruffle dominated, $D_c \propto \Delta \phi_2 D_{cA}$). For comparisons with the experiments it can be rather conveniently approximated as

$$D_c \approx \Delta W_c \Delta \phi_2 + \Delta \phi_2 D_{cA} \sin^2 \alpha \approx 4(\Delta \phi_2 \sin^2 \alpha + 0.88 \Delta W_c e^{-\frac{1}{0.88 \Delta W_c} \Delta \phi_2}) \left( \frac{\Delta \phi_2}{\Delta W_c} \right),$$

(1.5)

or $D_c \approx 4(\Delta \phi_2 + 0.88 \Delta W_c e^{-\frac{1}{0.88 \Delta W_c} \Delta \phi_2})$.

Fig. 4. Combined neoclassical ripple transport coefficient $D_c$ versus the normalized ruffle strength. The dashed line shows derived approximation.

Figure 5 shows the measured expansion rate $\nu_{\langle r^2 \rangle}$ for the case $\sin^2 \alpha_1 = 1$ as a function of ruffle voltages $\pm \Delta V_2$ at the wall. It has essentially the same fitting function as in Fig. 4, giving the normalized “radially averaged” ruffle strength as $\langle \Delta \phi_2/\Delta W_c \rangle \approx (4/3) \Delta V_2/1V$, which is close to its calculated value. Thus, at $B = 6$ kG and $\Delta V_2 = 3$ V the effective ruffle width $\Delta \phi_2 \approx 4 \Delta W_c$, and the transport rate has changed by $4 \times$ accordingly.

$\nu_{\langle r^2 \rangle} \approx 0.11(\Delta V_2 + 0.66 e^{-\frac{\Delta V_2}{0.66V}})$

Fig. 5. Measured expansion rate as a function of the ruffle voltage $\Delta V_2$ at the wall.

Figure 6 is a plot of measured expansion rate $\nu_{\langle r^2 \rangle}$, taken during step-by-step rotation of the magnetic tilt orientation angle $\theta_B$ for various tilt strengths $\varepsilon_B$ at the fixed wall ruffle $\Delta V_2 = 1.1$ V. The ruffled-induced part shows an unambiguous $\sin^2 \alpha$ dependence on relative angle $\alpha = \theta_B - \theta_2$ (here, $\theta_2 \approx \pi/4$) with magnitude proportional to $\varepsilon_B^2$; and varying $\theta_2$ in steps of $\pi/2$ (not shown) verifies the dependence on relative angle only.

Figure 7 is a plot of measured expansion rate $\nu_{\langle r^2 \rangle}$ versus magnetic tilt orientation angle $\theta_B$ for various applied wall ruffle strengths $\Delta V_2$, now at the fixed tilt strength $\varepsilon_B = 0.001$. Once again, the ruffled-induced part shows unambiguous $\sin^2 \alpha$ signature, but now with magnitude proportional to $\Delta V_2$.
The distinctive $\varepsilon_B^2 \sin^2 \alpha$ signature, together with separate control of $\Delta V_2$ and $\varepsilon_B$, enables experimental identification of neoclassical transport processes separately from $z$-kinetic processes. We model the full transport as

$$v_{c\gamma} = C_{CA}(\Delta V_2)\varepsilon_B^2 + C_{CA}(\Delta V_2)\varepsilon_B^2 \sin^2 \alpha + C_{K1} \Delta V_2 + v_{c\gamma}(bkg),$$

where $C_{CA}$ and $C_{CA}(\Delta V_2)$ represent the radial integrals of Eqn. (1.4); $C_{K1}$ and $C_{K2}$ represent collisional (bounce-resonant) transport driven by $\varepsilon_B^2$ and $\Delta V_2^2$ as $\varepsilon$-dependent "error" fields [12-14]; and small $v_{c\gamma}(bkg)$ arises from uncontrolled background tilts, separatrices, and omnipresent ruffles. Here, for dimensional simplicity, $\varepsilon_B \equiv \varepsilon_B/(1 \text{ mRad})$ and $\Delta V_2 \equiv \Delta V_2/(1 \text{ Volt})$.

$$\Delta V_2 = 1.1V \quad V_{eq} = 6V \quad B = 6kG$$

Fig.8. Measured $\varepsilon_B^2$ scalings for the $C_{CA}$ and $C_{CA}(\Delta V_2)$ neoclassical helical ripple transport terms at $B = 6kG$. Every marker here (not shown for $C_{CA}(0)$) is the result of $(a + b \sin^2 \alpha)$ fit as those shown in Fig.6.

$C_{CA}$ and $C_{CA}(\Delta V_2)$ are readily obtained from the $\sin^2 \alpha$ dependences as those shown in Figs.6 and 7, and varying $\varepsilon_B$ gives the expected $\varepsilon_B^2$ scaling as the one shown in Fig.8 for $\Delta V_2 = 1.1V$ and $B = 6kG$ ($C_{CA} \approx 0.056/\text{sec}$). Data taken with $\Delta V_2 = 0$ define $C_{CA}(0) \approx 0.033/\text{sec}$; and just by comparing it to $C_{CA}(1.1V) \approx 0.019/\text{sec}$ and using the $D_{CA}(\Delta \phi_2/\Delta W_r)$ data from Fig.2, one can get another estimate on the "radially averaged" ruffle strength as $\langle \Delta \phi_2/\Delta W_r \rangle \approx (4/3)\Delta V_2/1V$, which is consistent with the previous conclusion based on the results in Figs.4 and 5.

Data taken with $\varepsilon_B = 0$ show a $v_{c\gamma}^{(bkg)}$ offset and a parabolic dependence on a varied $\Delta V_2$, giving $C_{K2}$. Varying $\varepsilon_B$ then selects $C_{CA}$ and $C_{K1}$; these terms are distinguished by their $B$-scaling (discussed next), and by the fact that the $\varepsilon$-antisymmetric bounce-averages in $C_{CA}$ require the separatrix, whereas the kinetic $C_{K1}$ depends only weakly on the applied squeeze voltage. In Fig.7, $C_{K2}(4kG) \approx 0.03$, giving elevated $\sin^2 \alpha$ minima for large $\Delta V_2$; the depressed minima for $\Delta V_2 = 0.33$ are from ruffle-suppression of $D_{CA}$ (see Fig.2); and $v_{c\gamma}^{(bkg)} \approx 0.007/\text{sec}$.

Figure 9 shows the measured transport rates $C_{CA}$, $C_{CA}$ and $C_{K1}$ versus magnetic field with empirical scalings (dashed), compared to theory (solid lines). At high $B$, the chaotic and collisional separatrix transport processes agree closely with theory, scaling as $B^{-1}$ and $B^{-1/2}$ respectively. Here the accuracy of comparison is limited by temperature uncertainty, sensitivity to edge density gradients, and induced modification of $F_M(\phi_0)$. At low $B$, the kinetic transport labeled $C_{K1}$ is observed to depend strongly on field ($\propto B^{-2.7}$), but no simple power law is expected theoretically as bounce-rotation resonances become dominant. Prior scaling experiments have been confused by the presence of uncontrolled separatrices and ruffles, as well as by overlapping transport regimes [8].

Similar enhanced particle transport is observed when there are temporal variations in the separatrix energy. Figure 10 illustrates the immediate increase in radial expansion rate induced when white noise ($V_{rms} = 0.2 V$, $f_k < f < 20 MHz$) is applied to the $\theta$-symmetric ripple (squeeze) ring, driving randomly enhanced collisionless separatrix crossings. The $\dot{\varepsilon}$ increase in $(d\varepsilon/\varepsilon)^2$ rate observed here is consistent with a collisional separatrix layer width $\Delta W_r \approx 0.07 \varepsilon V$ fluctuating by $\Delta \phi(\theta) \approx 0.2 \varepsilon V$. Presumably, any noise- or wave-induced fluctuations which change particle kinetic energies relative to the separatrix energy would be equally effective in enhancing neoclassical helical ripple transport.

**4. CONCLUSIONS**

Most plasma confinement devices have trapping separatrices (ripples), arising from variations in magnetic field strength or external potentials. These separatrices are never perfectly symmetric, or perfectly aligned with other asymmetries. If the separatrix itself is asymmetric or temporally perturbed, the drifting particles collisionlessly change from trapped to passing and back, leading in the case of low collisionality ($v_f < f_k \ll 1$) to enhanced asymmetric superbanana ripple transport ($\propto v_f B^{-1}$) in comparison to the standard neoclassical ripple transport ($\propto v_f^{1/2} B^{-1/2}$). When the separatrix layer collisional width is less than its $\theta$-asymmetry or temporal perturbations, this new loss mechanism becomes the dominant bulk transport process in our non-neutral plasma experiments.
and it could have important implications for similar low collisionality regimes in other magnetic confinement experiments.

![Enhanced expansion rate during two bursts of 0.2 V (RMS) white noise Δφ(t) applied to a \( V_{eq} = 6V \) electrostatic ripple](image)

**Fig.10. Enhanced expansion rate during two bursts of 0.2 V (RMS) white noise Δφ(t) applied to a \( V_{eq} = 6V \) electrostatic ripple**

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**REFERENCES**


**ЭКСПЕРИМЕНТЫ ПО НЕОКЛАССИЧЕСКИМ ПЕРЕНОСАМ НА ВОЗМУЩЕННЫХ СУПЕРБАНАНОВЫХ ТРАЕКТОРИЯХ В ЭЛЕКТРОННОЙ ПЛАЗМЕ**

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Гофрировка тороидального поля приводит к появлению локально-запертых частиц, и традиционная неоклассическая диффузия описывает процессы переноса обусловленные их кулоновскими столкновениями на супербанановых орбитах. В наших экспериментах с аксиально-несимметричной гофрировкой исследованы новые эффекты хаотического (бесстолкновительного) неоклассического переноса, как вызванные \( E \times B \) вращением электронной плазмы вдоль полоидально-возмущенных сепаратрис, так и на сепаратрисах возмущаемых волнями взаимодействиями (микрофлюктуациями) в плазме.