

# MHD Assumptions :

didactic : Longitudinal, Transverse  
or Irrotational, Solenoidal

$$\nabla \cdot E_Q = 4\pi \rho_Q = 0$$

$$\nabla \cdot B_M = 0 = 4\pi \rho_M \text{ (optional)}$$

$$c \nabla \times E_t = -\dot{B} \text{ photons}$$

$$c \nabla \times B = \cancel{E_t} + \dot{E}_Q + 4\pi J_Q + 4\pi J_t$$

cancel

$$F = \cancel{\rho_Q} E + (\cancel{J_Q} + J_t) \times B / c$$

$$\nabla \cdot J = -4\pi \cancel{\rho_Q} = 0$$

$\rho_Q = 0$  : No Charges  
 $E_Q = 0$  : No Elec.Pot.Egy  
 No Thermo.Elec  
 No Gravi.Elec  
 No Capacitance

$$\begin{Bmatrix} E \\ B \\ J \end{Bmatrix} \equiv \begin{Bmatrix} E_Q \\ B_M \\ J_Q \end{Bmatrix} + \begin{Bmatrix} E_t \\ B_t \\ J_t \end{Bmatrix}$$

$$\nabla \times \begin{Bmatrix} E_Q \\ B_M \\ J_Q \end{Bmatrix} = 0 \quad \nabla \cdot \begin{Bmatrix} E_t \\ B_t \\ J_t \end{Bmatrix} = 0$$

$J_t$  is "curling"  
or "spin" current

No Causality  
(Simultaneous)

No E // B

$J_Q = 0$  : No Particle Current

Induction of  $J_t$  only

" $\sigma_t$ " =  $\infty$  (Ideal)

?? Spin Currents do not dissipate

?? Moving B-lines live forever

?? B-lines "Frozen-Into" Plasma

?? Plasma "Stuck on" B-lines

?? Particle Streamline  $\equiv$  B-line

## Dissipation

$$v_{ei} = n \bar{v} \left(\frac{e^2}{T}\right)^2 \ln \Lambda$$

$$m \Delta v_{ei} v_{ei} = e E_Q \text{ (momentum)}$$

$$\sigma = \frac{e^2 n}{m v_{ei}} \approx (10^{14} \text{ s}^{-1}) T_{\text{eV}}^{3/2}$$

Hydro :  $v_{ei} \nearrow \Rightarrow T \searrow, \sigma \searrow$

Magnetic:  $\sigma \nearrow \Rightarrow T \nearrow, v_{ei} \searrow$

\* Contradictory

## MHD Assumptions :

One-fluid MHD describes a mathematical Fluid with Mass, but no Charge  $\rho_Q$ , which can nevertheless contains a disembodied (transverse) Current  $J_t$ , which creates the (transverse) Magnetic Field  $B_t$ , which then pushes on the Current and Mass .  
Connection to the physics of Maxwell-Lorentz is tenuous at best.

Eliminating Charge eliminates the longitudinal Electric Field  $E_Q$  and Potential  $\phi$ , as well as the Current  $J_Q$  from the motion of charges.

This eliminates all Electrostatic Potential Energy, as well as all Thermo-Electric and Gravi-Electric effects, and the eliminates the dynamics of Capacitance from charge position.

The much-discussed “No E parallel to B” follows from this mutilation of Maxwell-Lorentz.

The subtle “time-scale” assumption that  $\dot{E}_t$  is negligible eliminates Photons, Time-retardation, and Causality, enabling “tractable,” instantaneous solutions everywhere, given “knowledge” of  $J_t$  everywhere. Nature’s ubiquitous entropic Dissipation was “Ideally” eliminated in the transition from particles to fluid, and assuming  $J_t$  to be dissipation-less enables B-field “lines” to “live forever”, while “frozen-into” the fluid which is “stuck-on” the lines themselves.

For validity, MHD should require strong local particle collisionality to justify the fluid approximation, and weak current resistivity to justify the “ideal” assumptions.

However, plasma-particle resistivity  $\rho_d$  and collisionality  $v_c$  vary *oppositely* with plasma temperature; and  $\rho_d$  depends *only* on temperature T and the fundamental (particle) constant  $r_e$  .

These contradictory physical requirements make “the MHD regime” somewhat ill-defined.