Geometric Maxwell-Lorentz : Discrete e^-, p^+

$$\{\hat{E},\hat{B}\} = \{E,B\}/e \sim [l^{-2}]$$

$$\nabla \bullet \hat{E}(x) = 4\pi \sum_{j=1}^{N} s_j \,\delta(x - x_j) \qquad s_j = \pm 1$$

$$\nabla \bullet \hat{B}(x) = 0$$

photons

$$c\nabla \times \hat{E} = -\dot{\hat{B}}$$

$$c\nabla \times \hat{B} = \dot{\hat{E}} + 4\pi \sum_{j=1}^{N} s_j \mathrm{v}_j \,\delta(x - x_j)$$

$$a_k = e^2 \frac{s_k}{m_k}\left[\hat{E} + \left(\frac{\mathrm{v}_k}{c}\right)\times \hat{B}\right]$$

$$\mathcal{E} = \frac{e^2}{8\pi}\left(\hat{E}^2 + \hat{B}^2\right)$$

Fundamental Constants

t, l | c ; m | m_e

$$e^2 = 1.44 \text{ eV}\cdot\text{nm}$$

$$hc = 1240.\text{ eV}\cdot\text{nm}$$

$$r_e \equiv \frac{e^2}{m_e c^2} = 2.82\text{x}10^{-15}\text{ m}$$

or

$$\lambda\!\!\!^{-}_c \equiv \frac{\hbar c}{m_e c^2} = 386.\text{x}10^{-15}\text{ m}$$

$$a_0 = \lambda\!\!\!^{-2}_c / r_e \qquad r_e / \lambda\!\!\!^{-}_c = 1/137.$$

$$\hat{\mu}_B = \lambda\!\!\!^{-}_c / 2 \qquad \frac{G m_p^2}{e^2} = 0.81\text{x}10^{-36}$$

Plasma Parameters :

Size scale r_e , n

Rate scale $\beta \equiv \bar{\mathrm{v}} / c$

$$\bar{\mathrm{v}}^2 \equiv T / m_e$$

$$\Omega_{ce} \equiv eB/mc = r_e \hat{B} c$$

$$\omega_p^2 \equiv 4\pi n e^2 / m = n\, r_e\, 4\pi c^2$$

$$\lambda_D^{-2} \equiv 4\pi n e^2 / T = n\, r_e \beta^{-2} 4\pi$$

$$\text{"}\beta_p^{-1}\text{"} \equiv \frac{B^2}{8\pi nT} = \frac{r_e \hat{B}^2 \beta^{-2}}{n\; 8\pi}$$

$$V_A^2 \equiv \frac{B^2}{4\pi n m_p} = \frac{r_e \hat{B}^2}{n}\frac{c^2 m_e}{4\pi m_p}$$

Fluidize : $N \to N/\varepsilon$

$\varepsilon \to 0$

$$\{\hat{E},\hat{B}\} \to \{\hat{E},\hat{B}\}/\varepsilon$$

$$e \to \varepsilon e$$

$$m \to \varepsilon m$$

$$T \to \varepsilon T$$

$$r_e \to \varepsilon r_e \to 0$$

$$\rho_d \to \varepsilon \rho_d \to 0$$

Constant :

$\{E, B\}$

Q, $\mathcal{E}$

M, $\bar{\mathrm{v}}$

nT

MHD : $Q = 0$

$E = 0$

$\nabla \bullet J = 0$

Fluidize: $\to 0$

$$b \equiv e^2/T = r_e \beta^{-2} \to 0$$

$$\nu_c = n \bar{\mathrm{v}} \mathrm{b}^2 \, ln\Lambda = n\, r_e^2 \beta^{-3} \, ln\Lambda \to 0$$

$$\rho_d = \frac{m_e}{e^2}\frac{\nu_c}{n} = r_e \beta^{-3} \, ln\Lambda / c \to 0$$

$$g \equiv \frac{1}{n\lambda_D^3} = [n r_e^3 \beta^{-6} (4\pi)^3]^{1/2} \to 0$$

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In geometric units, $\{\hat{E},\hat{B}\}$ have dimension length^{-2} .
The fundamental constant e^2 = 1.44 eV.nm sets the EM energy scale;
or equivalently, $r_e = e^2/m_e c^2 = 2.8 \times 10^{-15}$ m sets the (only) size scale .

The analogous quantum constant hc = 1240. eV.nm ,
or equivalently the Compton wavelength λ_c ,
then determine the Bohr radius a_0 and the Bohr magneton μ_B.

Plasma parameters are then determined by
by the size scale set by re, density n, and geometric fields $\{\hat{E},\hat{B}\}$,
and by the rate scale $\beta = \overline{v} / c$.

The fluid approximation "infinitely partitions" particles, keeping e/m constant;
but this makes $r_e = 0$, eliminating the only scale size in the equations, and
eliminating the physical basis for collisions, resistive dissipation, and correlation.
With separate e- and p+ fluids, dissipation can be re-inserted ad-hoc.

Magneto-Hydro Dynamics extends this mutilation of the Maxwell-Lorentz equations,
by eliminating all charge density, and all non-inductive electric fields,
retaining only a disembodied inductive (solenoidal, curling) current.